Lecture 7: Dynamics on and of Networks

Complex Systems 530

How to generate networks?

- Real world networks (static or dynamic)—lots of network data out there
- Random networks!
  - Many of these can be used either as
    - static networks to run dynamics *on*, or
    - models of dynamics *of* networks

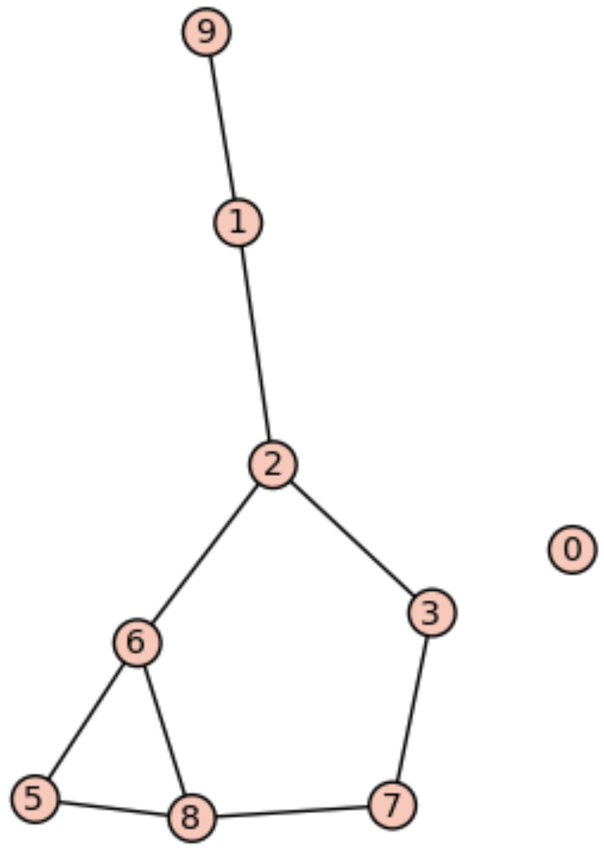
Random Networks

- Why would you want to do this?
  - Often want to simulate network formation or simulate dynamics on networks
  - May not know exact network
  - But often do know some general features of the network (e.g. degree distribution)
  - So: simulate random networks with those features

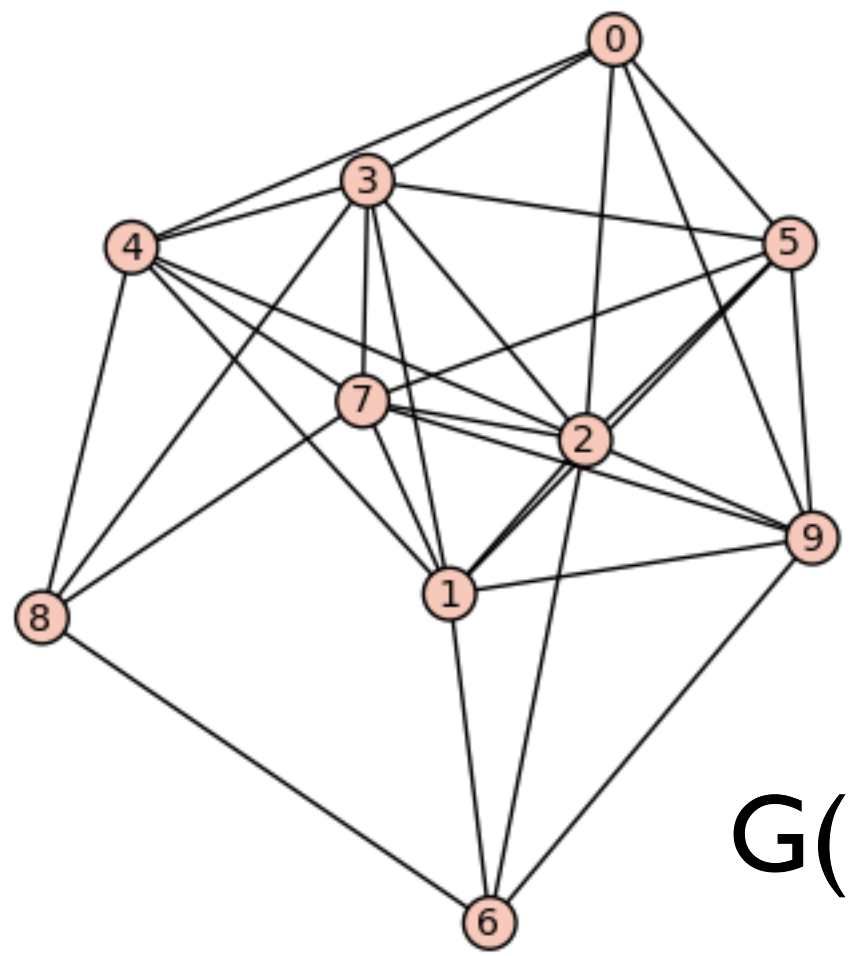
Erdös-Rényi Networks

- **Erdös–Rényi** (also Gilbert) Network - two forms:
  - *G(n,p)* - network on *n* nodes with each edge having probability *p* of existing
  - *G(n,M)* - network on *n* nodes with *M* edges chosen randomly
  - **Often called a "random graph"** even though all of the networks here are also random

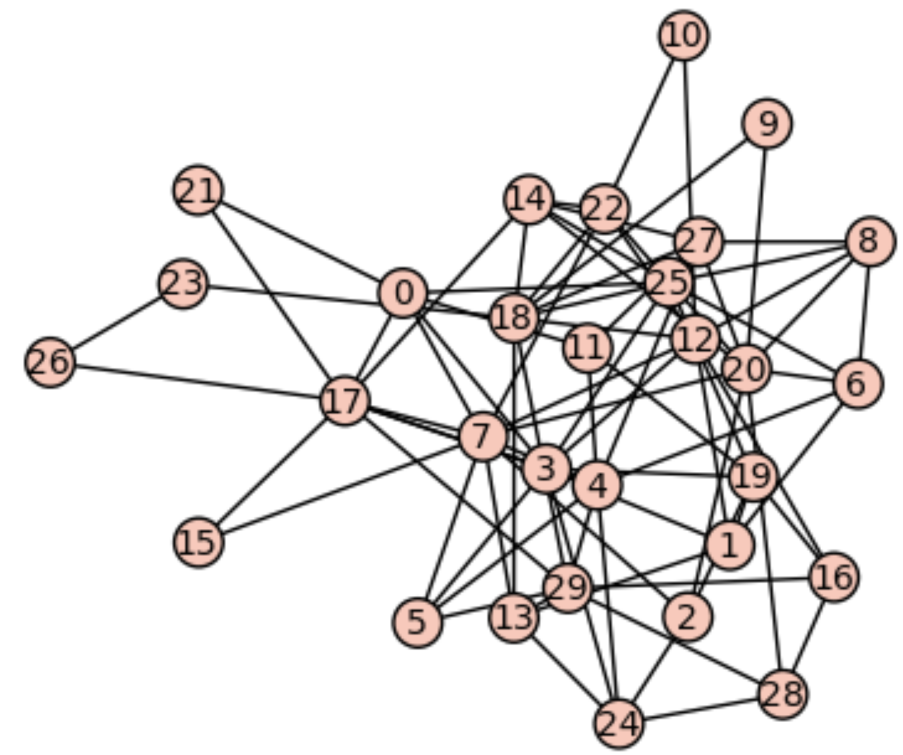
G(n,p)



G(10,0.2)

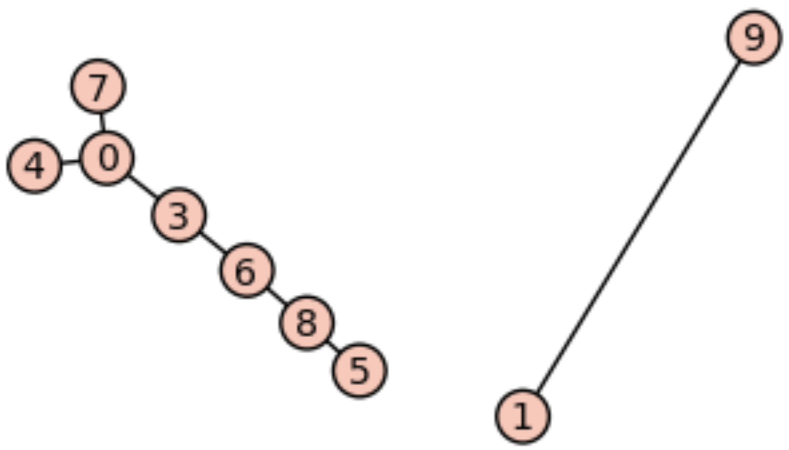


G(10,0.6)

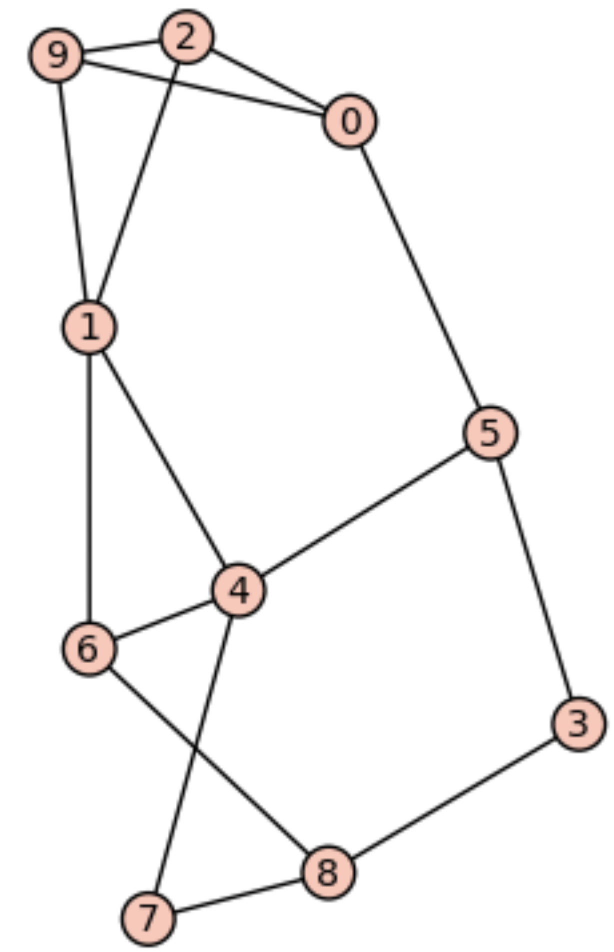


G(30,0.3)

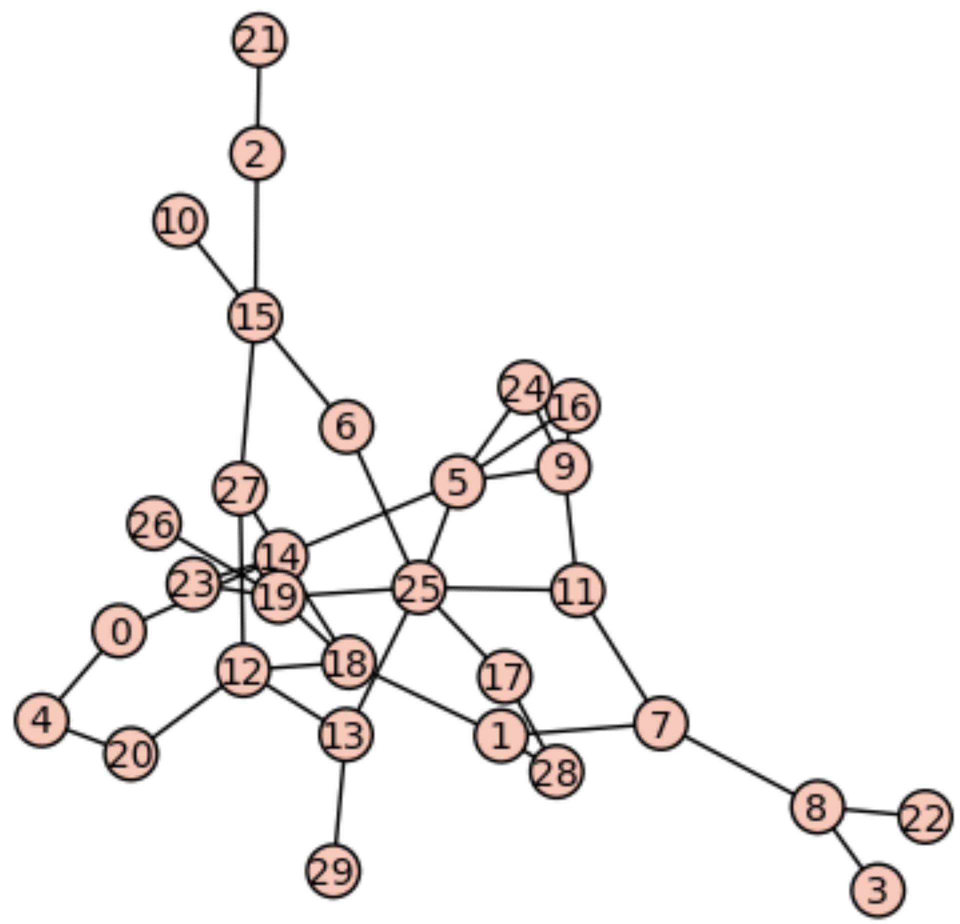
G(n,M)



G(10,7)



G(10,15)



G(30,40)

Erdös-Rényi Networks

- Not so realistic for lots of things (e.g. social networks, many gene/protein/biological networks)
- But, often handy as a test case/comparison point (e.g. if evaluating whether a mean-field model is a reasonable approximation)
- Useful for making analogs of homogeneous mixing (e.g. from SIR or compartmental models)

Erdös-Rényi Networks

- Let you sample from the full space of possible graphs with minimal assumptions

- If a property of a network is reproduced by ER, may suggest it's not a special feature of the network driving it—alternatively if ER does not reproduce this property, it may be more "interesting"

Erdös-Rényi Networks

- Lots of mathematical theory for random matrices (e.g. useful for examining adjacency matrices) and random graphs, particularly for Erdös-Renyi graphs, e.g.
  - Degree distribution, giant component, etc.

Milgram’s Small World Experiment

- Sent packages to random people in Wichita, Kansas
- Letter inside asked them to forward to a target person in Sharon, Massachusetts
- Told they could mail the letter directly to the target person only if they knew him personally, otherwise send it & instructions to a relative or friend they thought would be more likely to know the target person

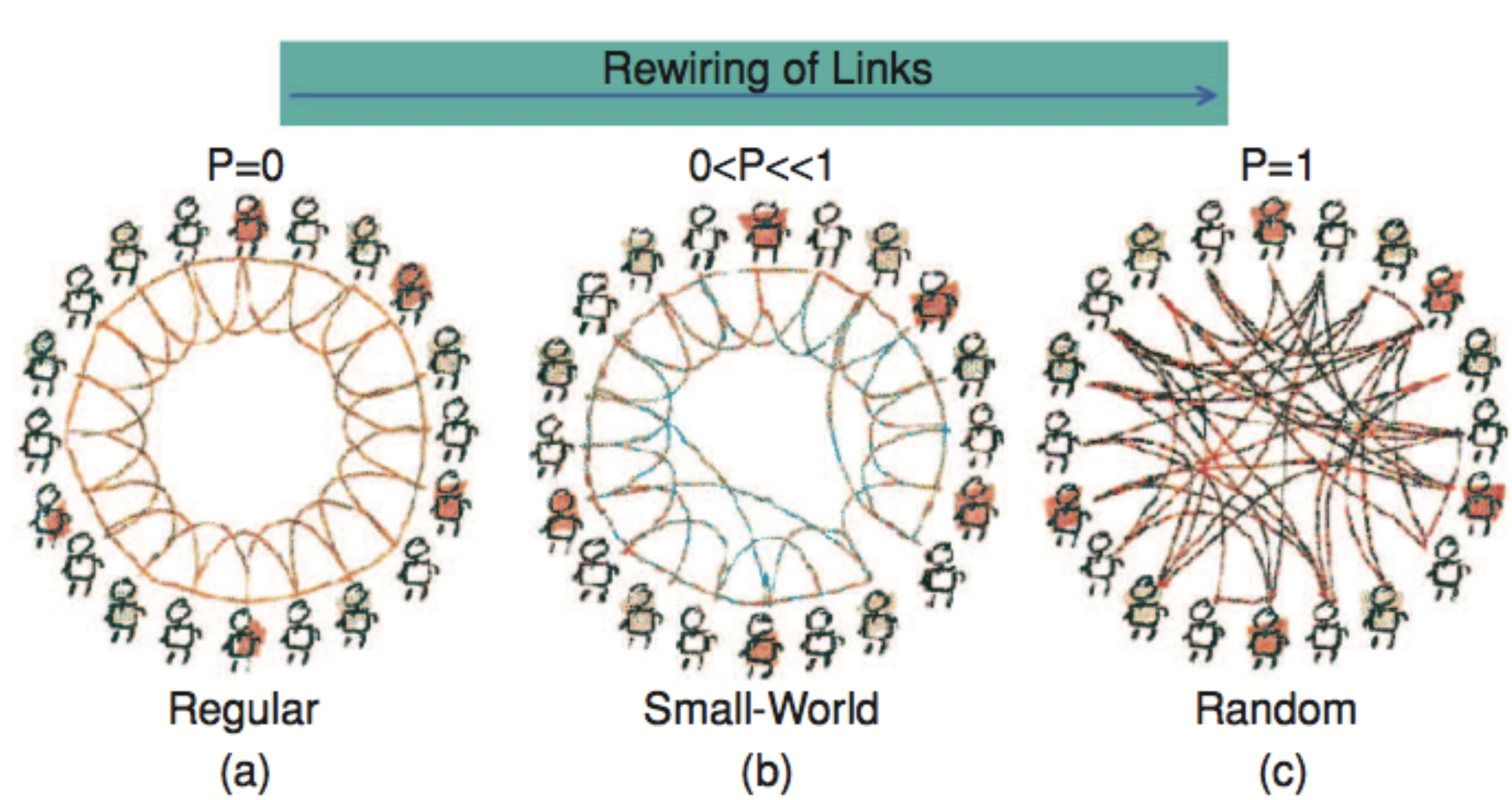
Milgram’s Small World Experiment

- Many letters didn’t make it, but among those that did, average path length was 6
- “Six degrees of separation”
- How to generate a small world network?

Small World Networks

- Regular graphs: clustered, but path length *L* grows linearly with number of nodes *n*
- Erdös-Rényi graphs: not clustered but small path length (grows as *log n*)
- Want to combine both

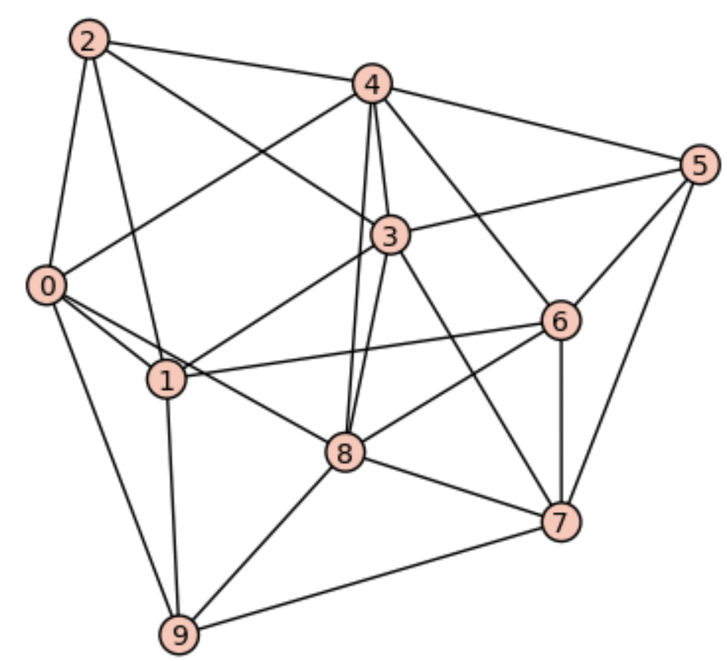
Newman-Watts-Strogatz Algorithm



Small World Networks

- Most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops or steps

- Average distance *L* between two nodes is proportional to *log n* (where *n* is the number of nodes)



Small World Network

- Creates the “what a small world!” effect: two nodes will tend to have a mutual friend (adjacent node)
- Can be similar to scale free in that can produce hubs as well as sparsely connected individuals
  - Network can be both small-world and scale-free
  - However, N-W-S tends to produce more similar degrees for nodes rather than scale free

Preferential Attachment Networks

- **Barabasi-Albert** algorithm

- Add new nodes to the network sequentially, preferentially connecting them to high-degree nodes

$$p(i) = \frac{\deg(i)}{\sum_j \deg(j)}$$

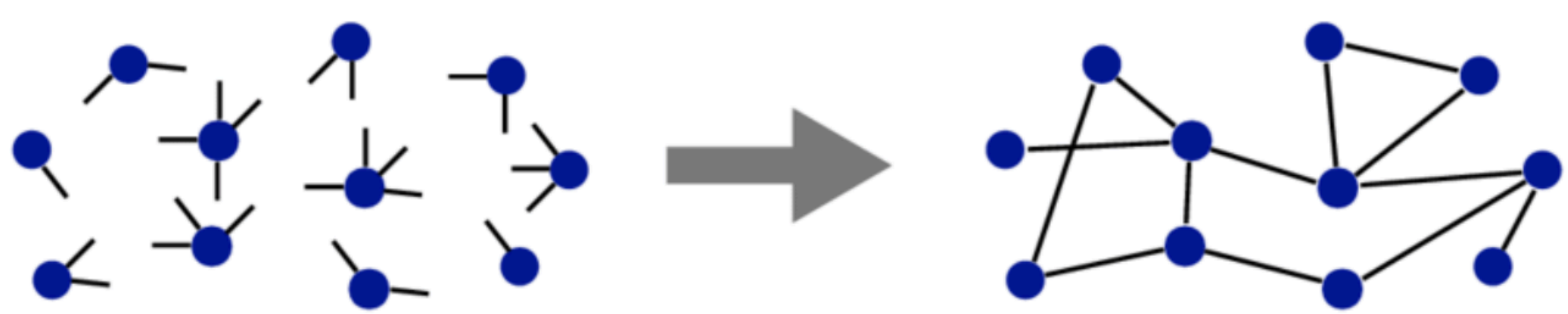
- Generates scale free networks

Preferential Attachment

- “Rich get richer” (Matthew effect) dynamics make hubs
- Can also implement as a growth process from an existing network

Configuration Models

- Given a degree sequence, generate random network with that sequence
- Random graphs, but with the advantage that the degree sequence can be chosen realistically
- Algorithm: generate ‘stubs’ with the correct degree, then connect pairs of stubs



Configuration Models

- Provides a way to generate random networks consistent with a real-world degree sequence/distribution
    - Often have non-network data that tells us about degree (egocentric data)
    - Or may want to explore the space of graphs that are ‘similar’ to a known network

Types of network dynamics

- **Dynamics on networks**: models where the processes of interest occur over a fixed network structure
- **Dynamics of networks**: models of the dynamic changes over time of the network topology itself
- **Adaptive networks**: models looking at the interplay of the two (both the processes on the network, and how the network changes)

Sayama, Chp. 16

Dynamics **on** Networks

- Dynamics on nodes and/or edges?
- What variables to consider?
  - Discrete vs. continuous variables
  - Deterministic vs. stochastic

Dynamics **on** Networks

- How to update?
  - Discrete vs. continuous time
  - Synchronous, asynchronous, continuous

Dynamics **on** Networks

- Discrete variable, discrete time—similar to CA! Just a different set of neighbors
  - Implementation is very similar
  - CA models are network models! Using a regular graph with a lattice structure with degree 4 or 8

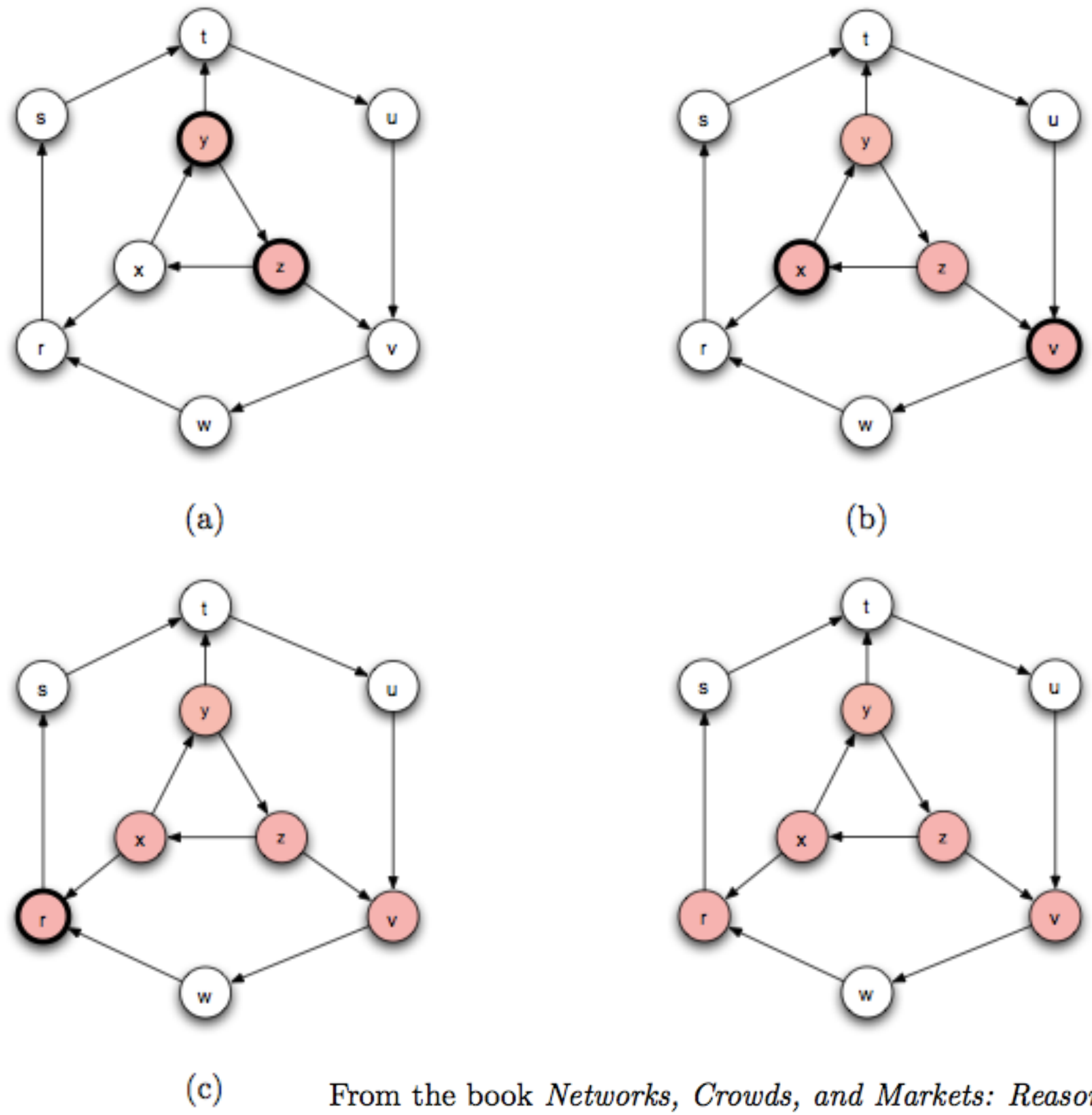
Example: infectious transmission on a network

- Infectious diseases, information/idea/culture propagation, behavioral dynamics (e.g. transmission of alcohol use behaviors)
- Nodes may be individuals, or they can be communities
- Edges indicate contact between individuals or communities, or potentially movement between communities

Example: infectious transmission on a network

- Each node may be assigned a status (susceptible/infectious/recovered)
- Or a vector/number (number of infected in that node, numbers of S/I/R in that node)
    - E.g. run an SIR model in each node but allow transmission within-node or between-node

Individual-level network models of disease transmission



From the book *Networks, Crowds, and Markets: Reasoning about a Highly Connected World.* By David Easley and Jon Kleinberg. Cambridge University Press, 2010.
Complete preprint on-line at http://www.cs.cornell.edu/home/kleinber/networks-book/

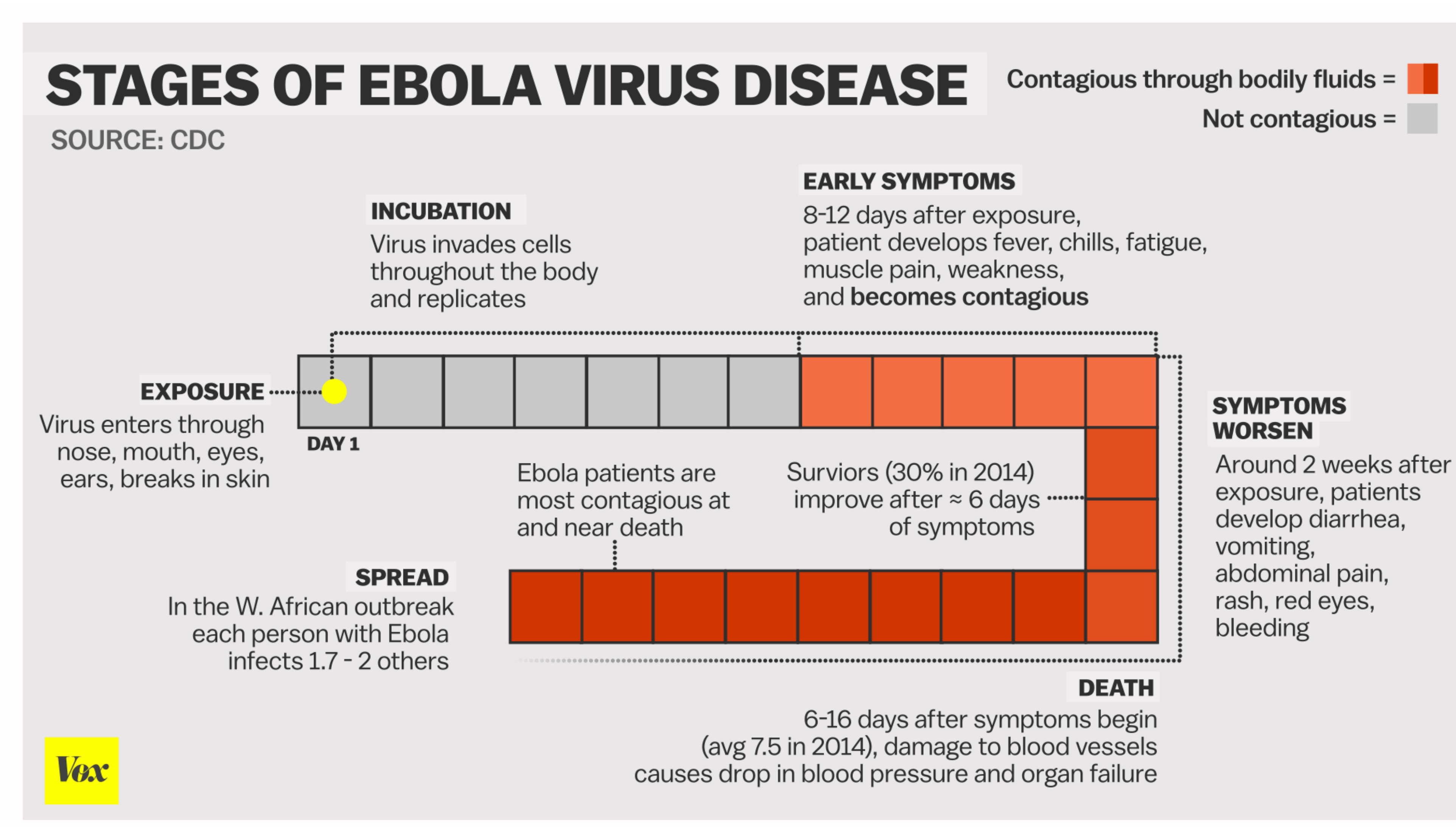
Individual-level network models of disease transmission

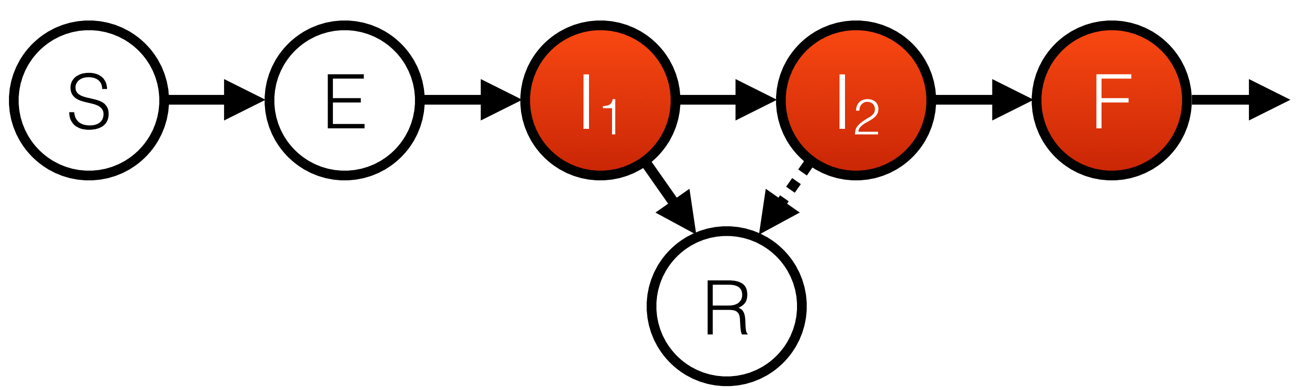
- Virus on a network example in NetLogo models library
- PyCX has several examples
- Let’s code one together!

Population level network model of disease transmission

- Can model population transmission on a network as an agent-based model or non-agent based model (e.g. ODE, stochastic model)

EVD in West Africa





D'Silva JP, Eisenberg MC. Modeling spatial invasion of Ebola in West Africa. Journal of theoretical biology. 2017 Sep 7;428:65-75.

Model Equations

$$\frac{dS}{dt} = -(\beta_I I_1 + \beta_2 I_2 + \beta_F F)S$$

$$\frac{dE}{dt} = (\beta_I I_1 + \beta_2 I_2 + \beta_F F)S - \alpha E$$

$$\frac{dI_1}{dt} = \alpha E - \gamma_1 I_1$$

$$\frac{dI_2}{dt} = \delta_1 \gamma_1 I_1 - \gamma_2 I_2$$

$$\frac{dF}{dt} = \delta_2 \gamma_2 I_2 - \gamma_F F$$

$$\frac{dR}{dt} = (1-\delta_1)\gamma_1 I_1 + (1-\delta_2)\gamma_2 I_2 - \gamma_R R$$

$$\mathcal{R}_0 = \frac{\beta_1}{\gamma_1} + \frac{\beta_2 \delta_1}{\gamma_2} + \frac{\beta_F \delta_1 \delta_2}{\gamma_F}$$

Measure: cumulative cases & deaths

Reporting Rate & Fraction of the Population at Risk

Fraction of individuals who have become infected x Population at risk x Reporting rate

= Observed cases

Reporting Rate & Fraction of the Population at Risk

Model

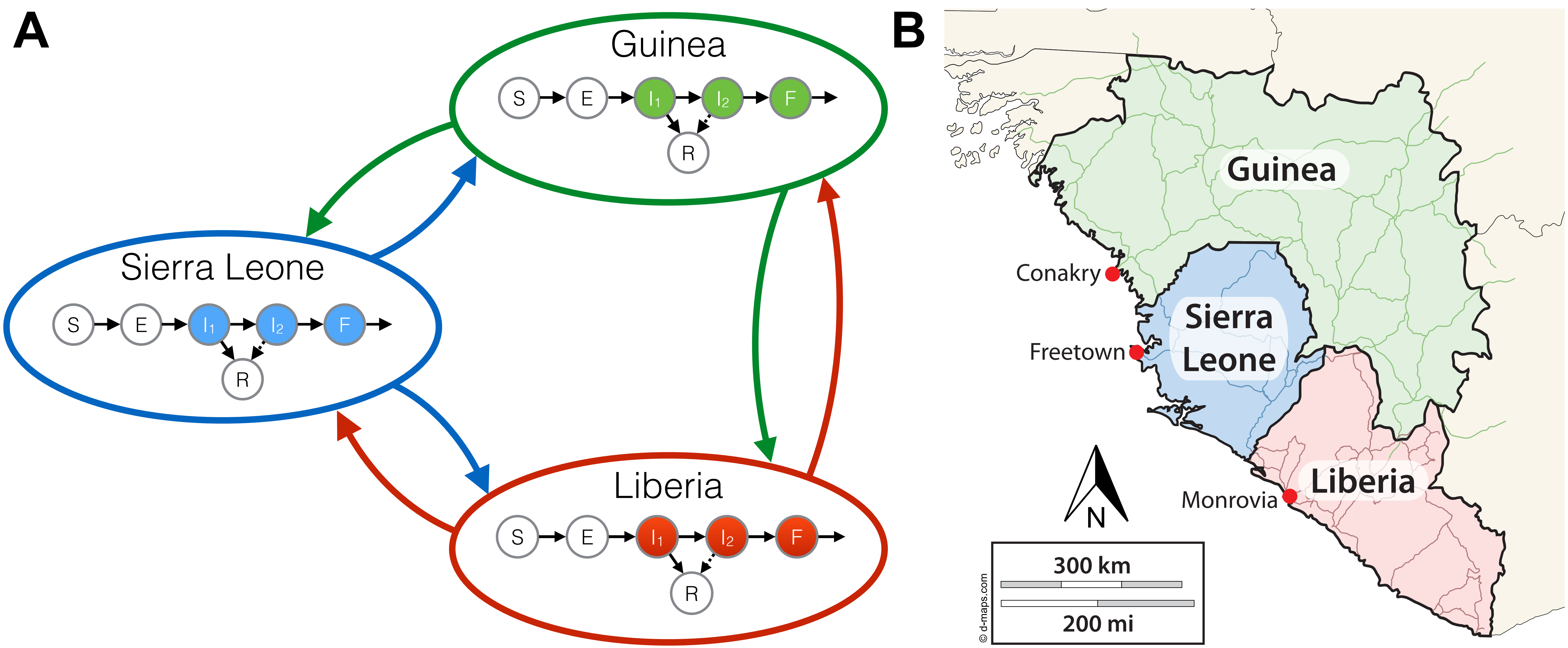
Fraction of individuals who have become infected x

k

Population at risk x Reporting rate

= Observed cases

Spatial network



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Gravity Model

- Model of transmission or movement between locations

- Suppose that contact is higher with regions that are larger (population centers), and regions that are closer

- Scale transmission or movement using ‘gravity’ term:

$$\theta_{ij} = \frac{N_i N_j}{d_{ij}^2}$$

$$\dot{S}_n = -(\lambda_n + \lambda_m + \lambda_l)S_n$$

$$\dot{E}_n = (\lambda_n + \lambda_m + \lambda_l)S_n - \alpha E_n$$

$$\dot{I}_{1n} = \alpha E_n - \gamma_n I_{1n} - r_{1,n} I_{1n}$$

$$\dot{I}_{2n} = \gamma_n I_{1n} - \delta I_{2n} - r_{2,n} I_{2n}$$

$$\dot{F}_n = \delta I_{2n} - \delta_2 F_n$$

$$\dot{R}_n = r_{1,n} I_{1n} + r_{2,n} I_{2n}$$

$$\dot{IC}_n = k_{norm} \alpha E_n$$

$$\dot{DC}_n = k_{norm} \delta I_{2n}$$

$$\lambda_n = \beta_{1,n} I_{1n} + \beta_{2,n} I_{2n} + \beta_{F,n} F_n$$

$$\lambda_m = \theta_{n,m}(\beta_{1,n} I_{1m} + \beta_{2,n} I_{2m} + \beta_{F,n} F_m)$$

$$\lambda_l = \theta_{n,l}(\beta_{1,n} I_{1l} + \beta_{2,n} I_{2l} + \beta_{F,n} F_l)$$

$$\theta_{n,m} = \kappa_n \frac{\rho_n \rho_m}{(d_{n,m})^{\iota}}$$

$$\theta_{n,l} = \kappa_n \frac{\rho_n \rho_l}{(d_{n,l})^{\iota}}$$

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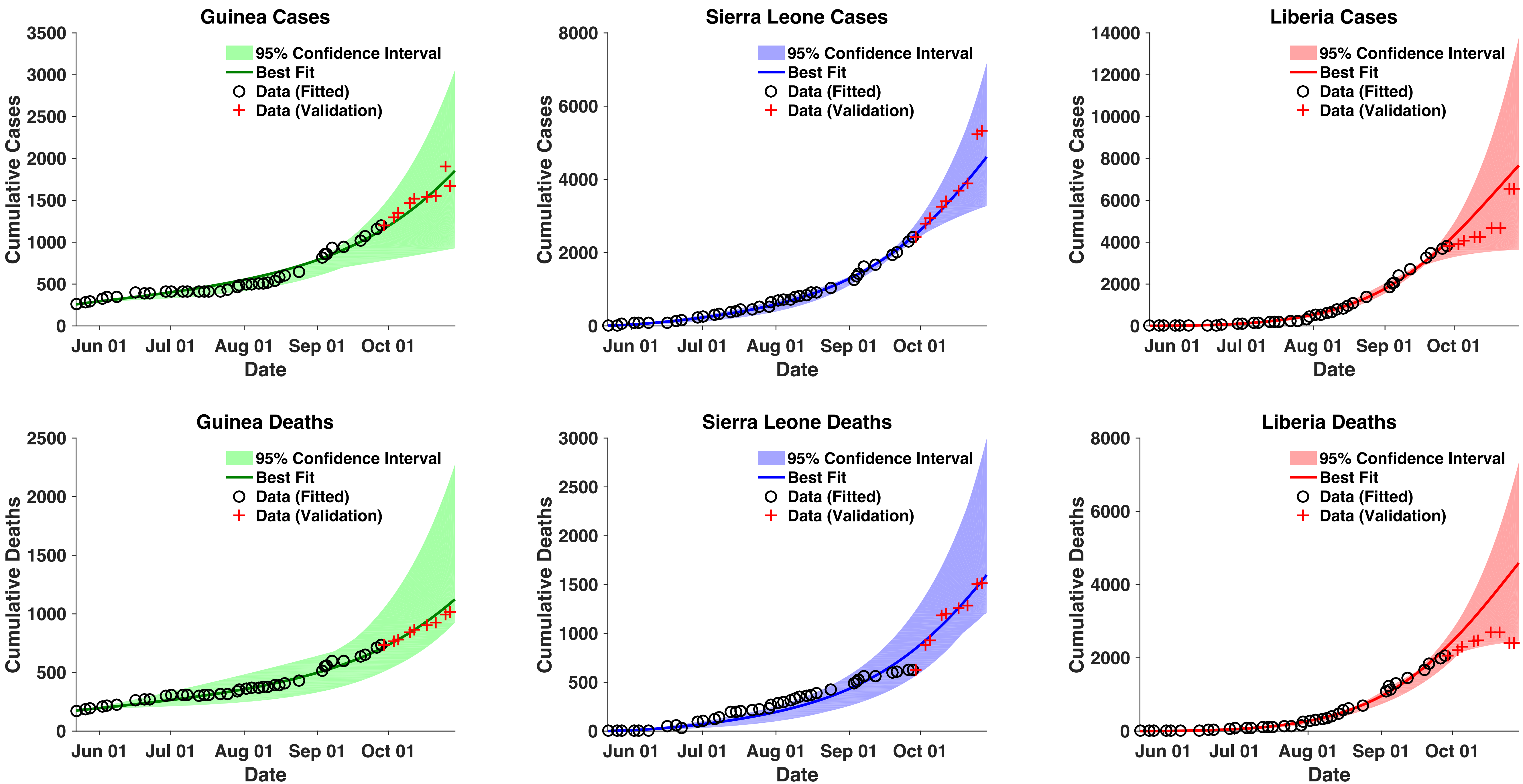
Parameter Estimation

- Estimate parameters from incidence data on cases and deaths

- Some parameter information from the literature and from ongoing reporting of incubation period, infectious period, etc.

- Extensive uncertainty and issues of unidentifiability!
    - Many different parameter values will fit the data equally (or close to equally) well



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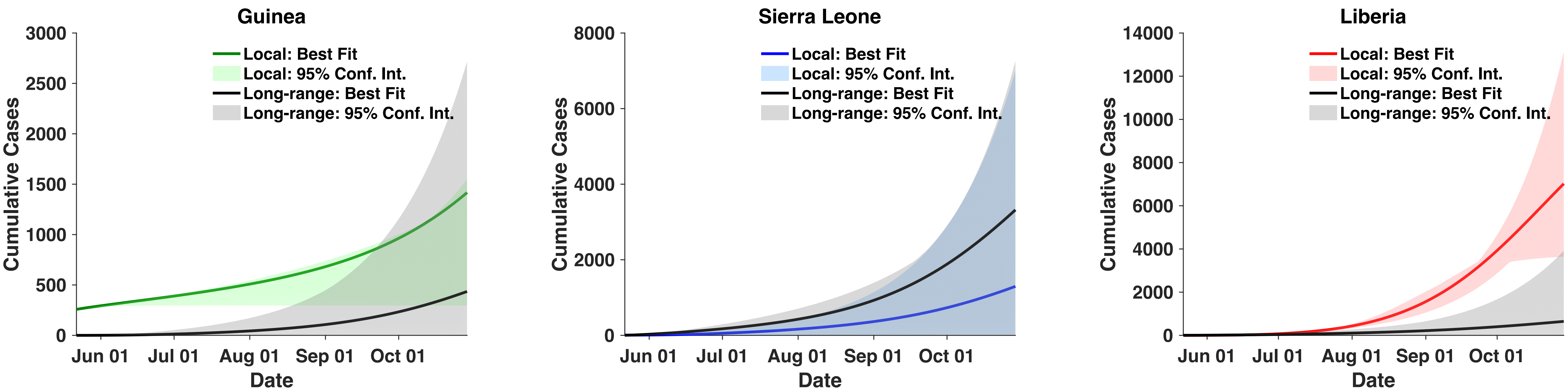

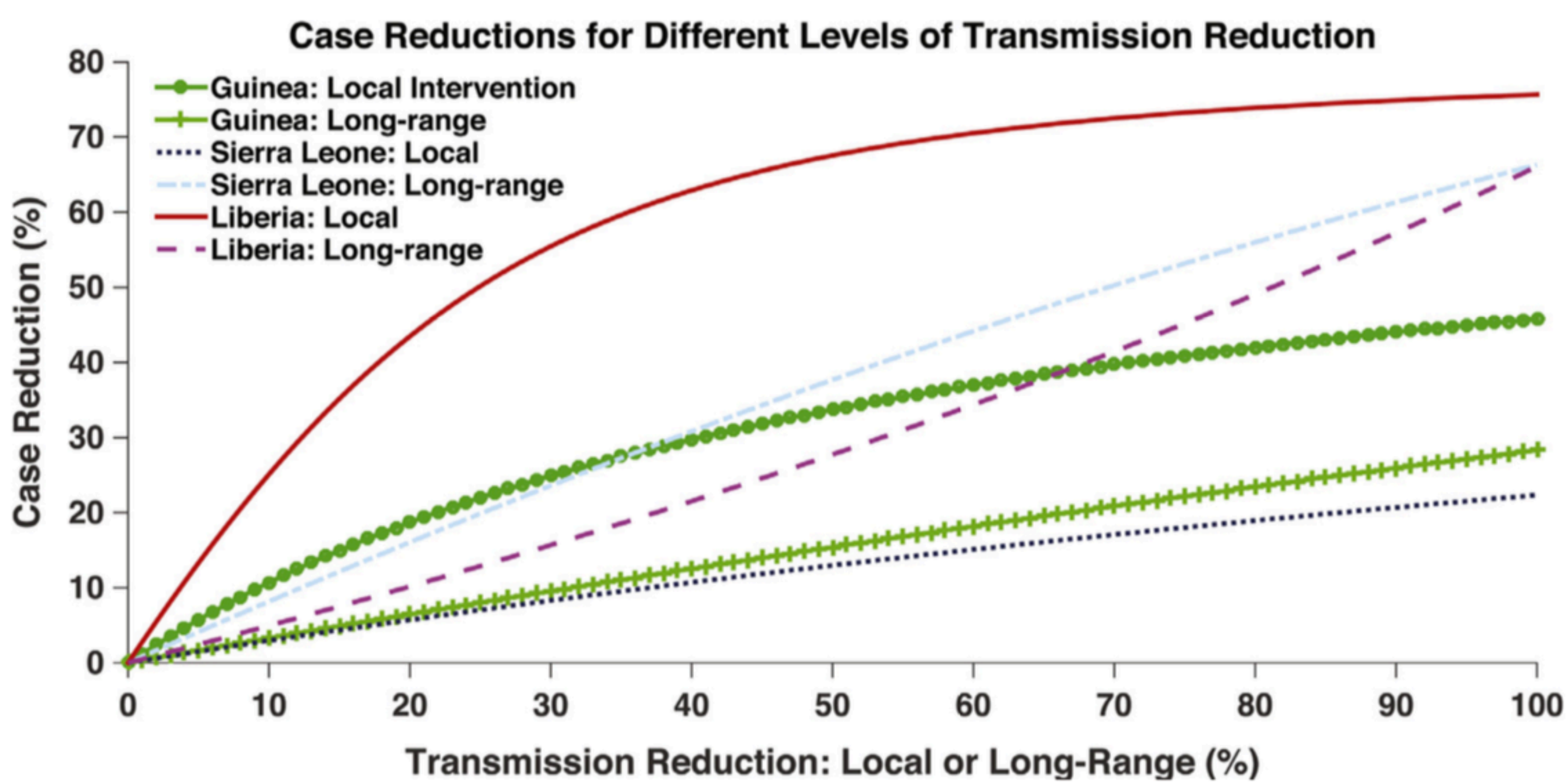

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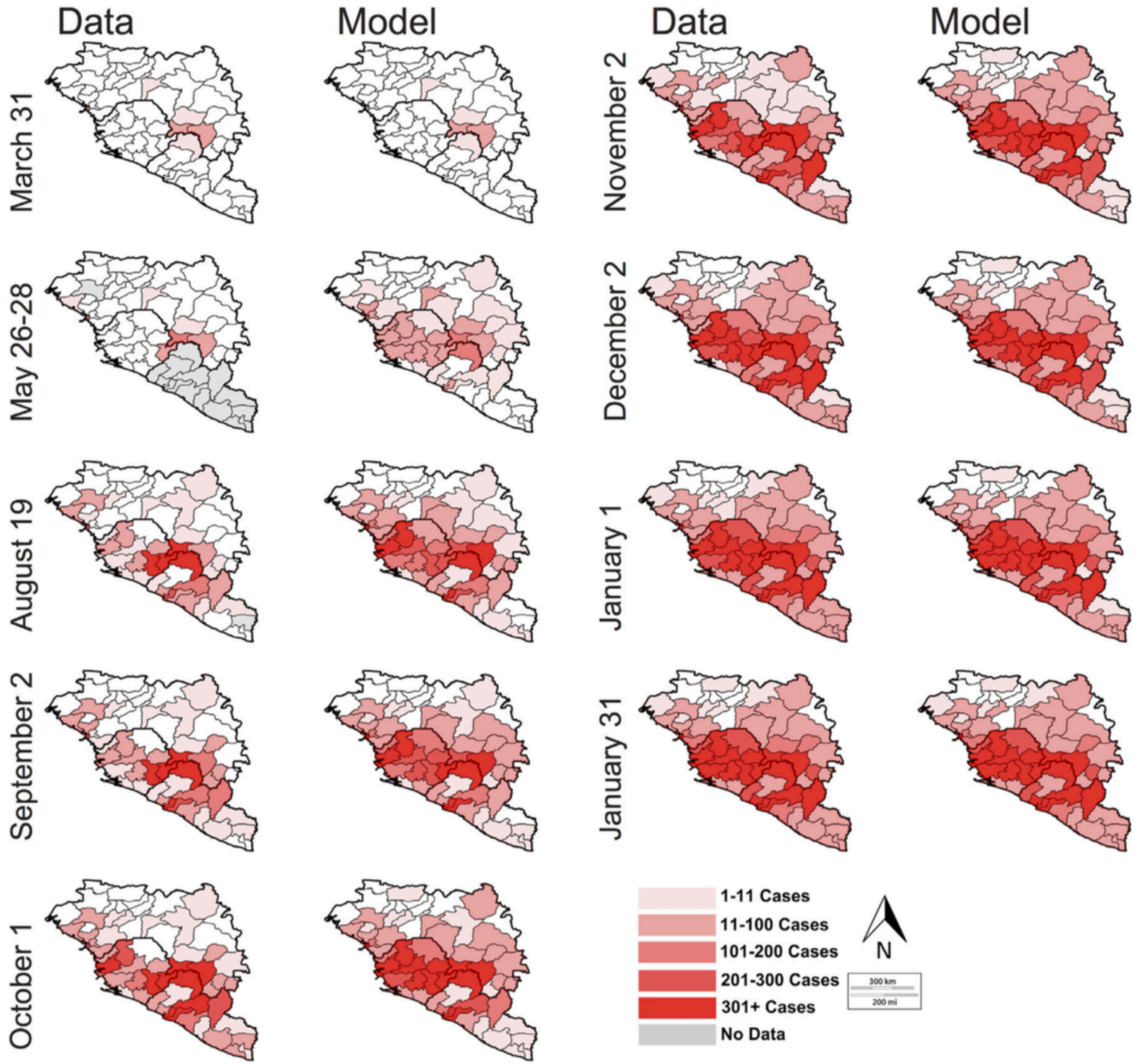
More granular: modeling at the district level

- Extend the model to the 63 districts in Guinea, Liberia, and Sierra Leone
- Adapt the model to be stochastic (since some districts have small population)



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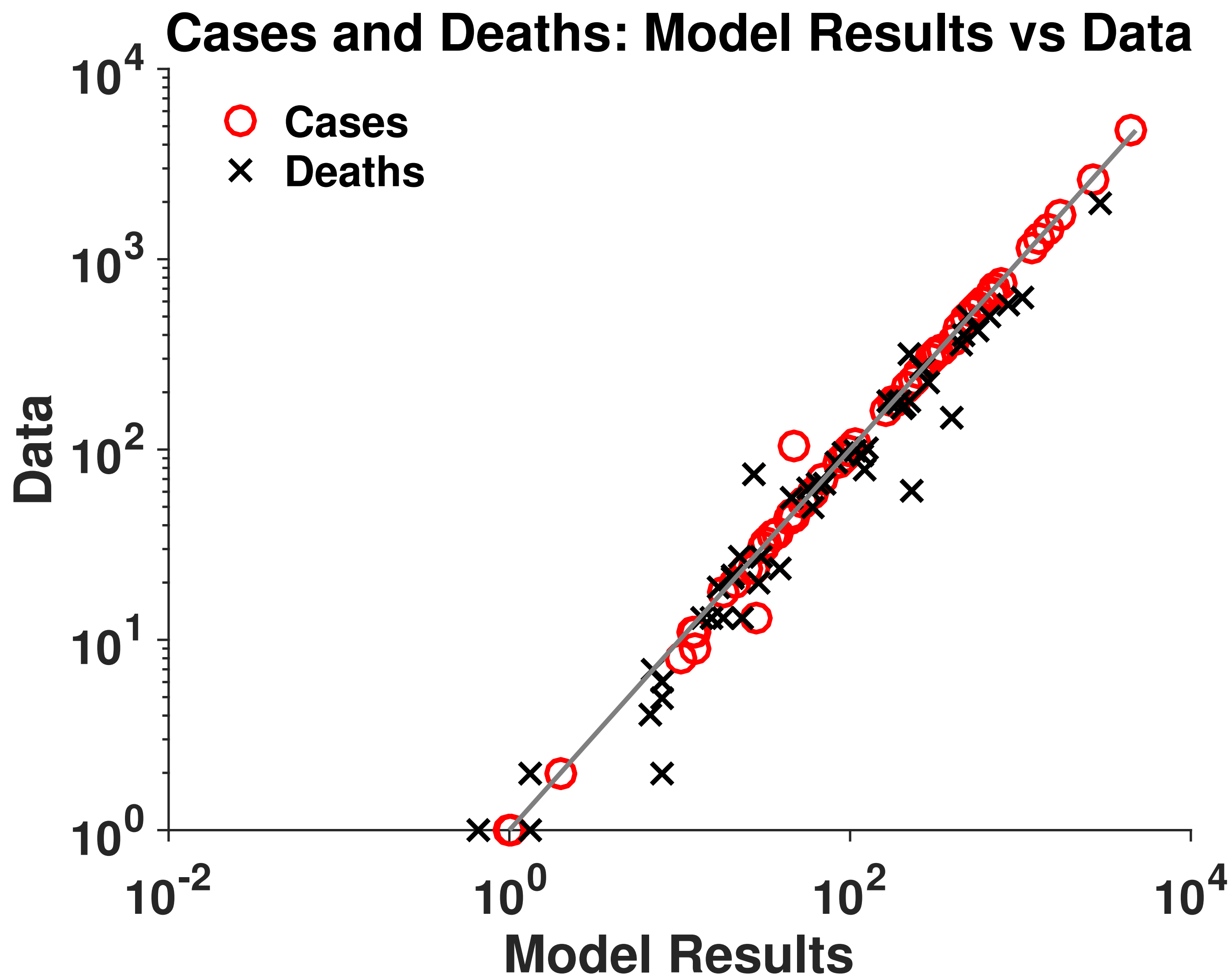

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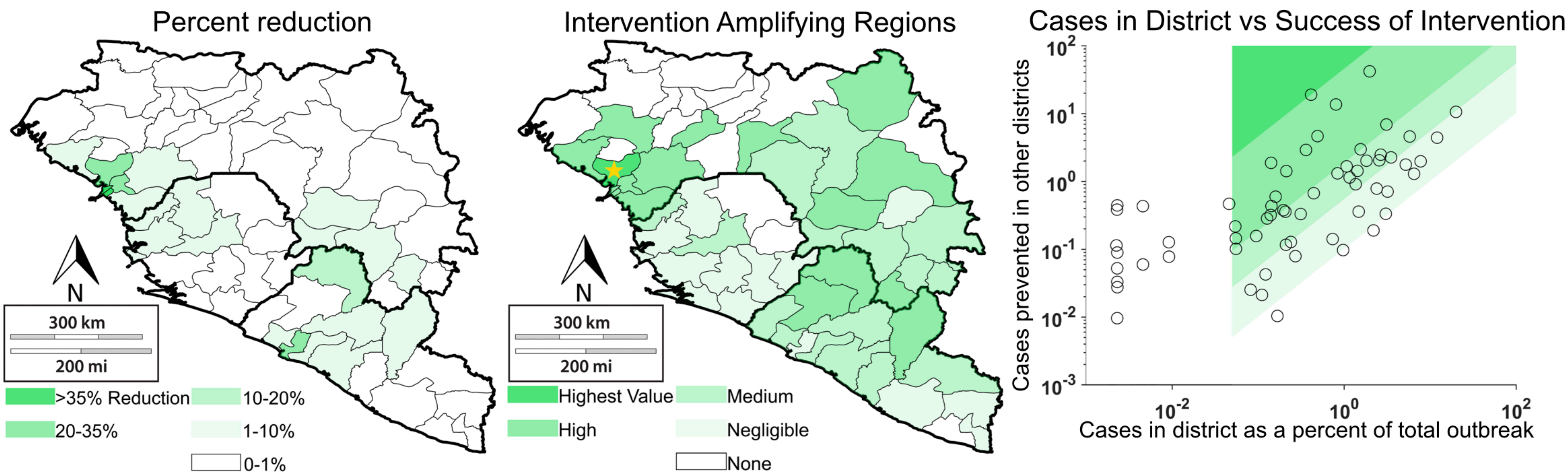

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Epidemic Dynamics on Networks

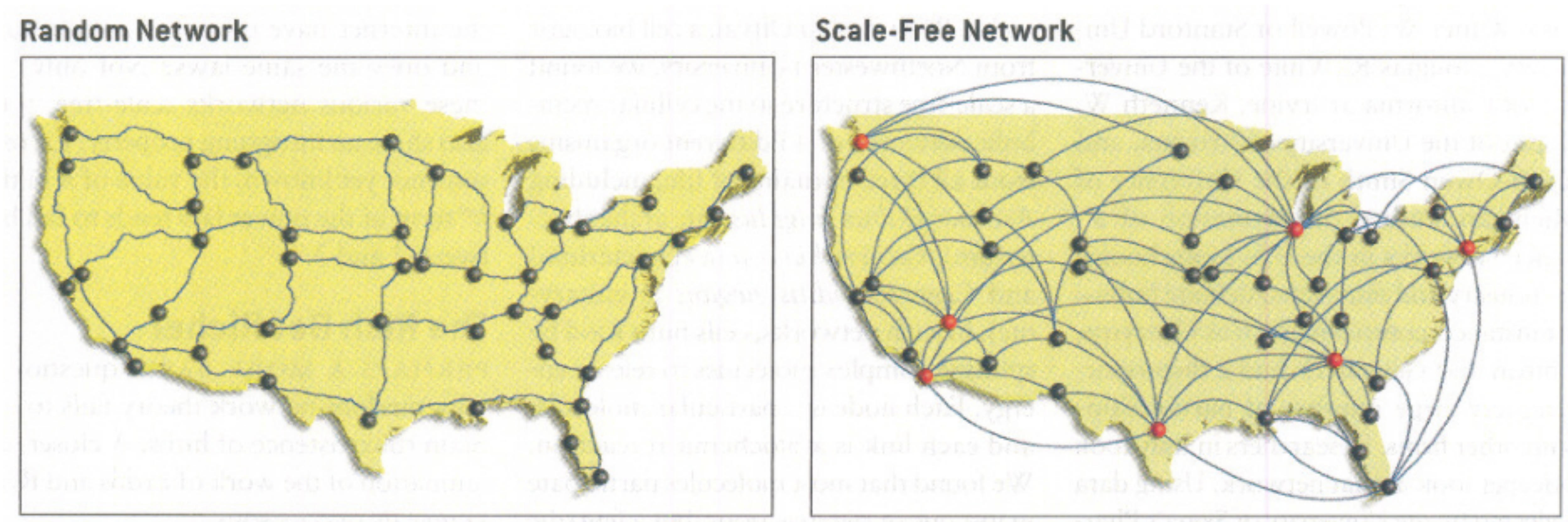
- Network structure plays a huge role on the epidemic dynamics
    - Hubs, sparsely connected, etc.
- Small world property can tend to produce synchronized epidemics (e.g. oscillations)

Epidemic Dynamics on Networks

- Where you place high-risk individuals or patches can significantly affect R_0, disease dynamics, etc.
  - E.g. if cluster high-risk nodes together vs spread apart
  - If hub vs periphery is infected - the scale free vulnerability to hub attacks

Epidemic Dynamics on Networks

- How would interventions/risk/dynamics differ for epidemic spread by roads vs air travel? (and what does this mean for pandemics & emerging diseases/behaviors)



Epidemic Dynamics on Networks

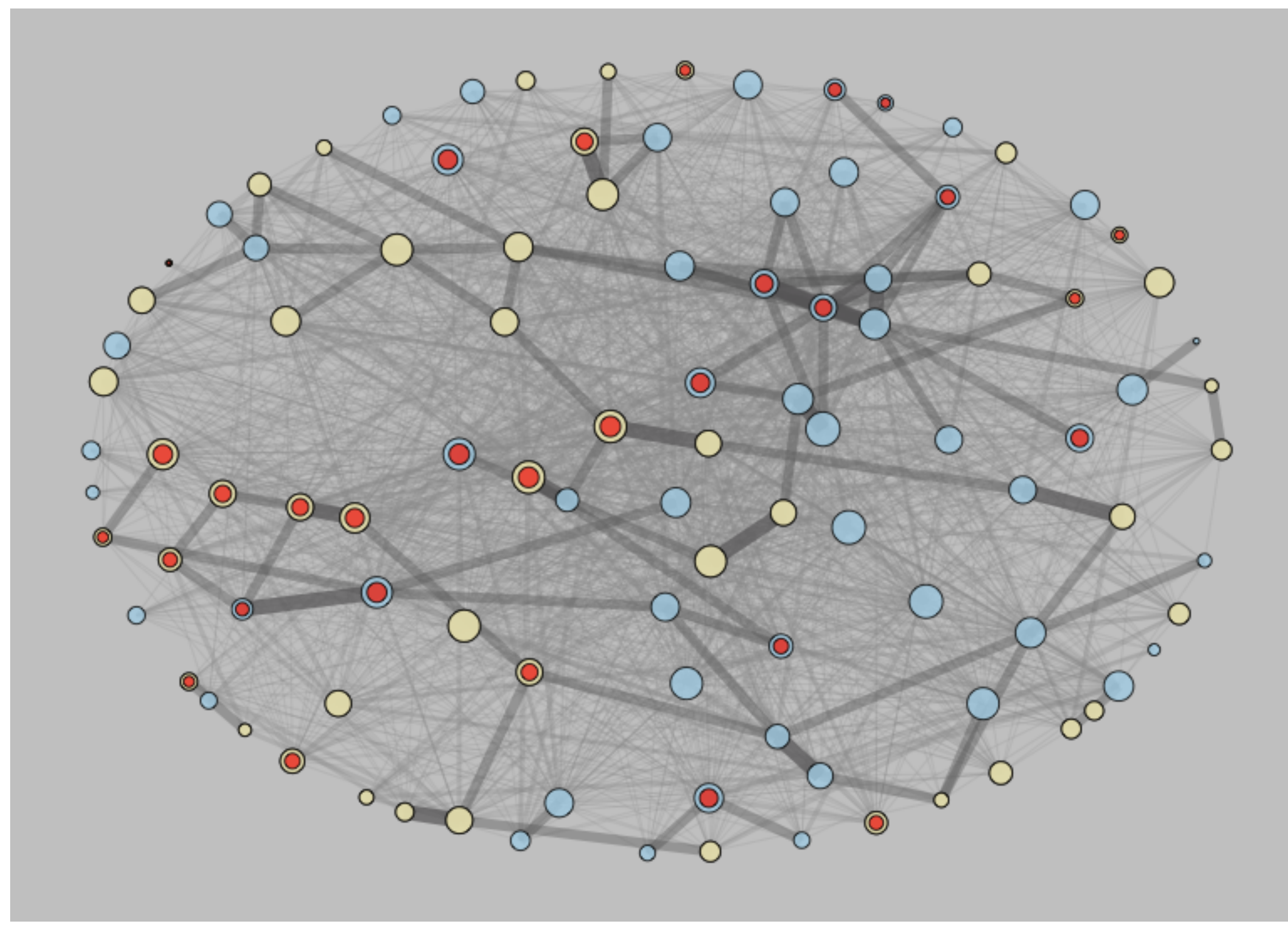
- Major & still very open area of research
- Can have significant impact on interventions & control strategies
  - Should you target well-connected individuals?
  - Are there specific network structures you should look for as high-risk?

Epidemic Dynamics on Networks

- Lots of interesting data to work with too—can often track contacts, etc.

- Example: the eX-FLU study (Aiello et al.)

- Substudy tracking contacts using Bluetooth from cell phones



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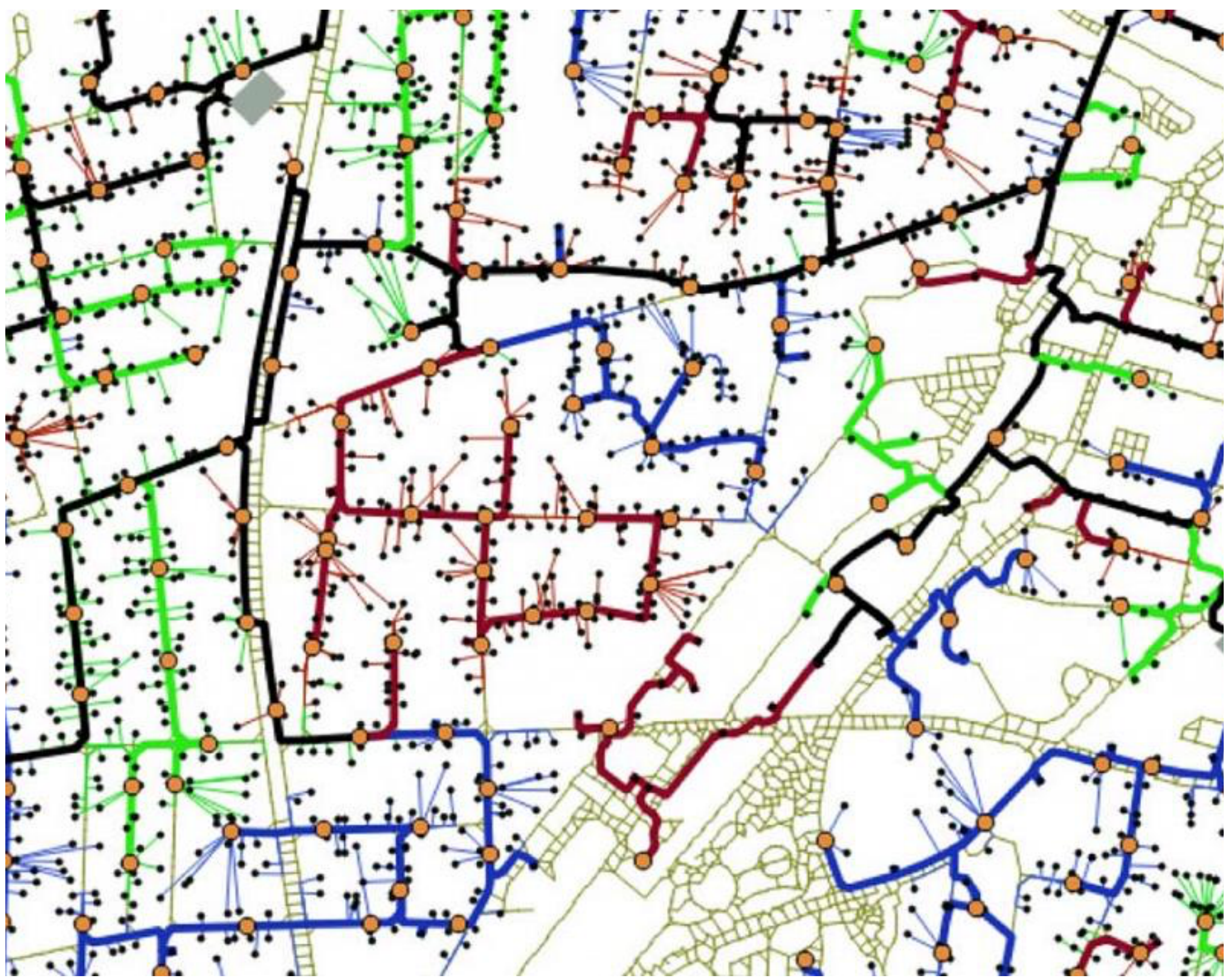
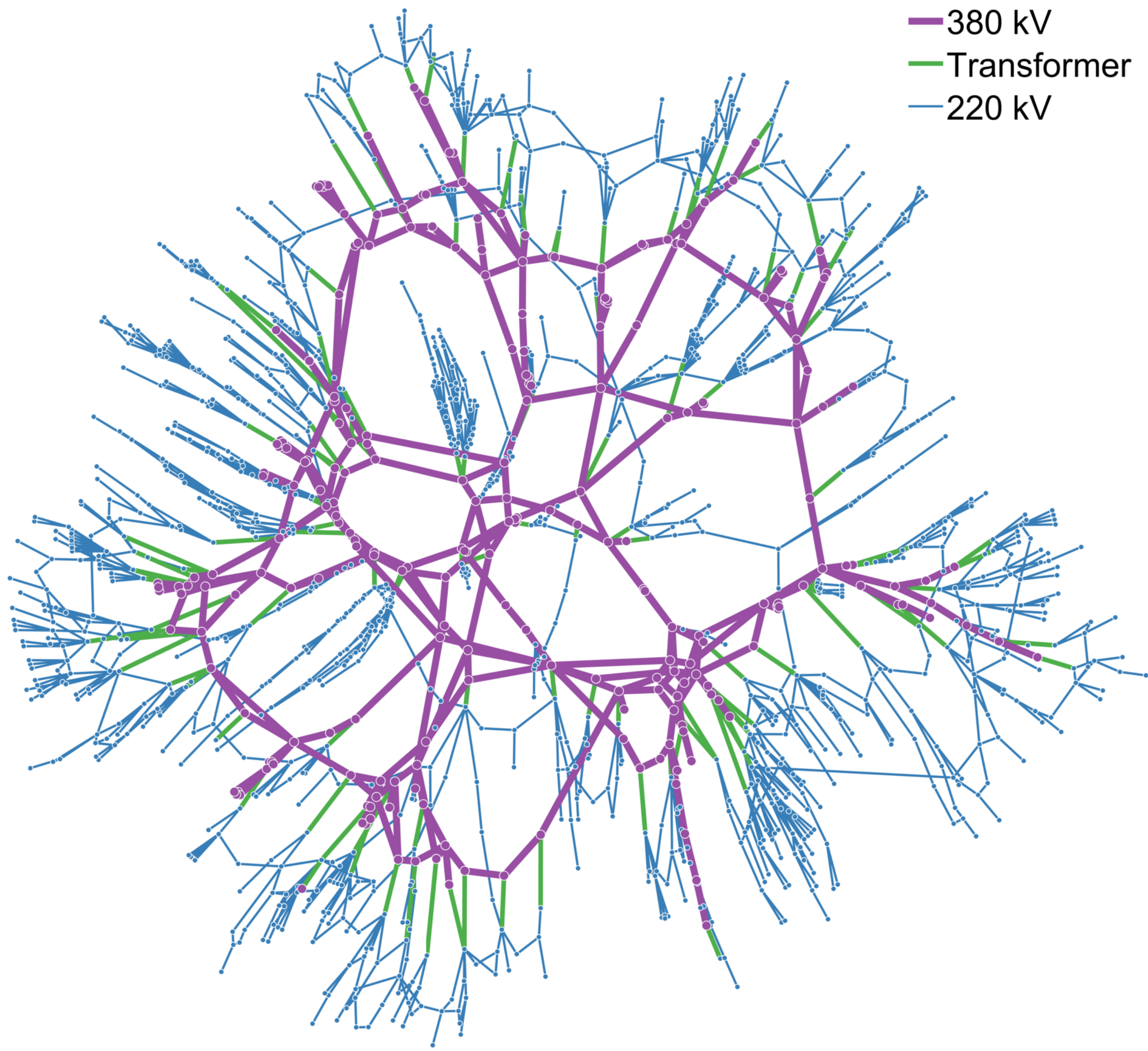
Example: power grids



SMART-DS: Synthetic Models for Advanced, Realistic Testing: Distribution Systems and Scenarios
https://www.nrel.gov/grid/smart-ds.html



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Example: Neuronal networks

- Firing dynamics on networks used extensively in mathematical/ computational neuroscience

- Example: **ring model of direction sense!**

- Proposed as a model in the 1990’s—this mechanism has since been found!

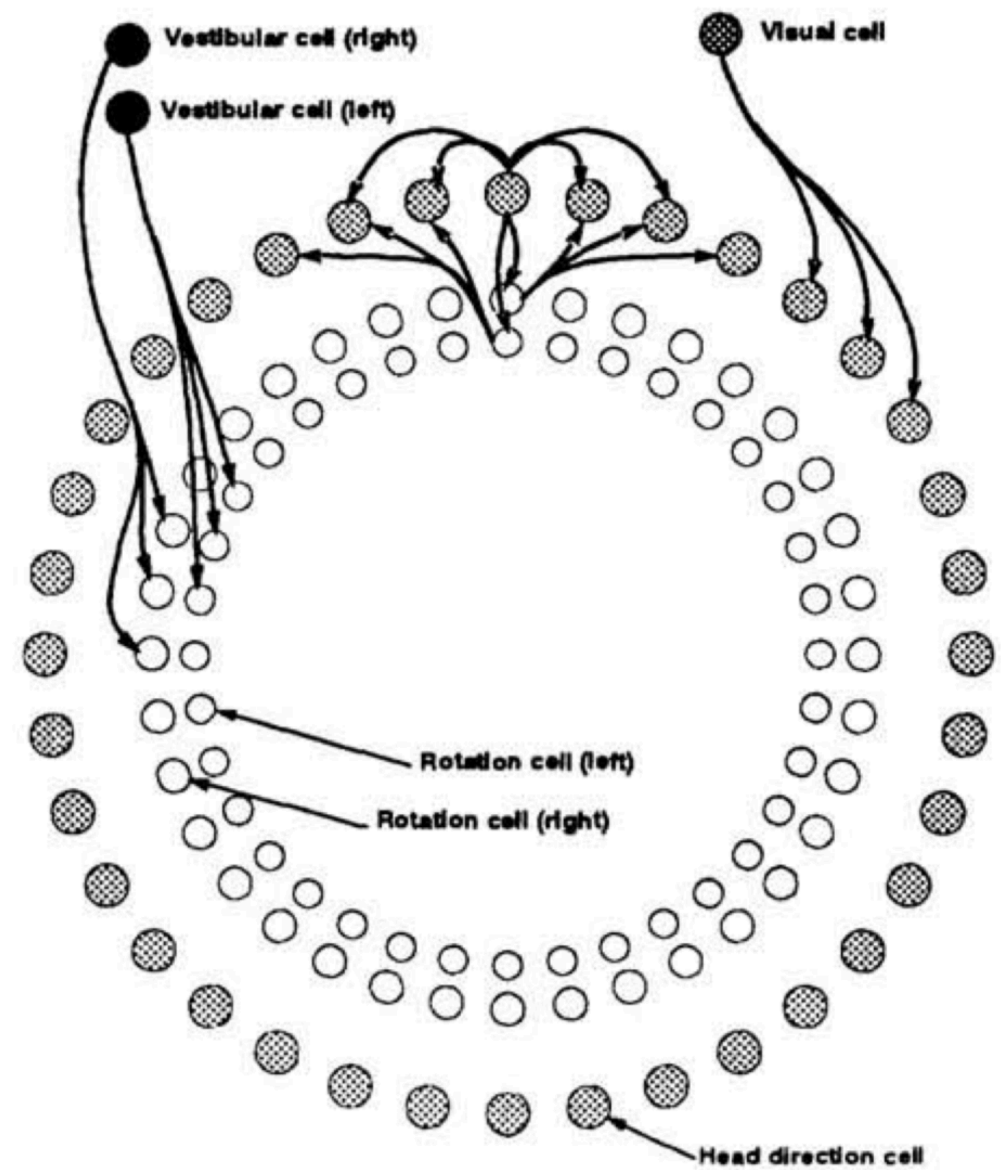
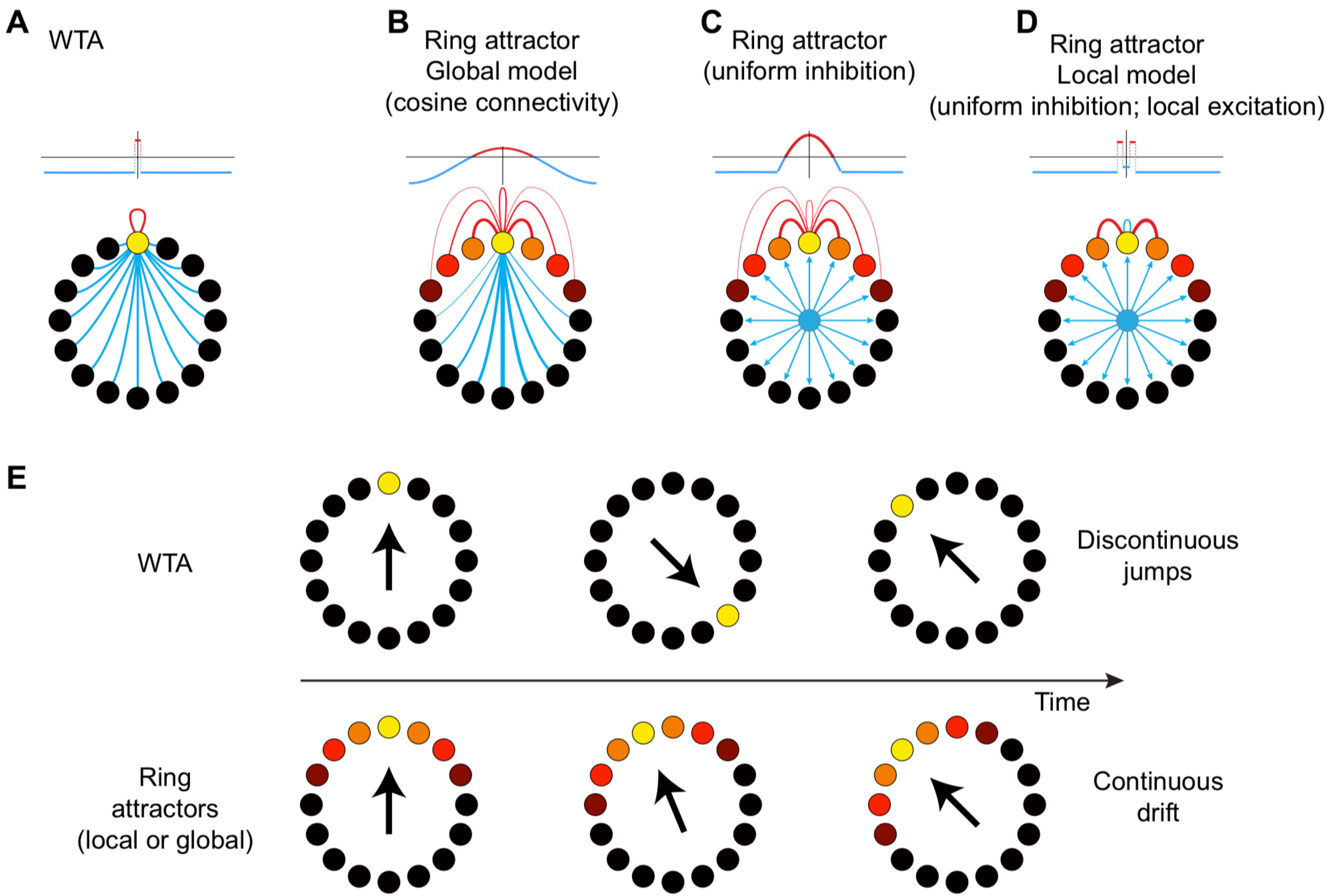


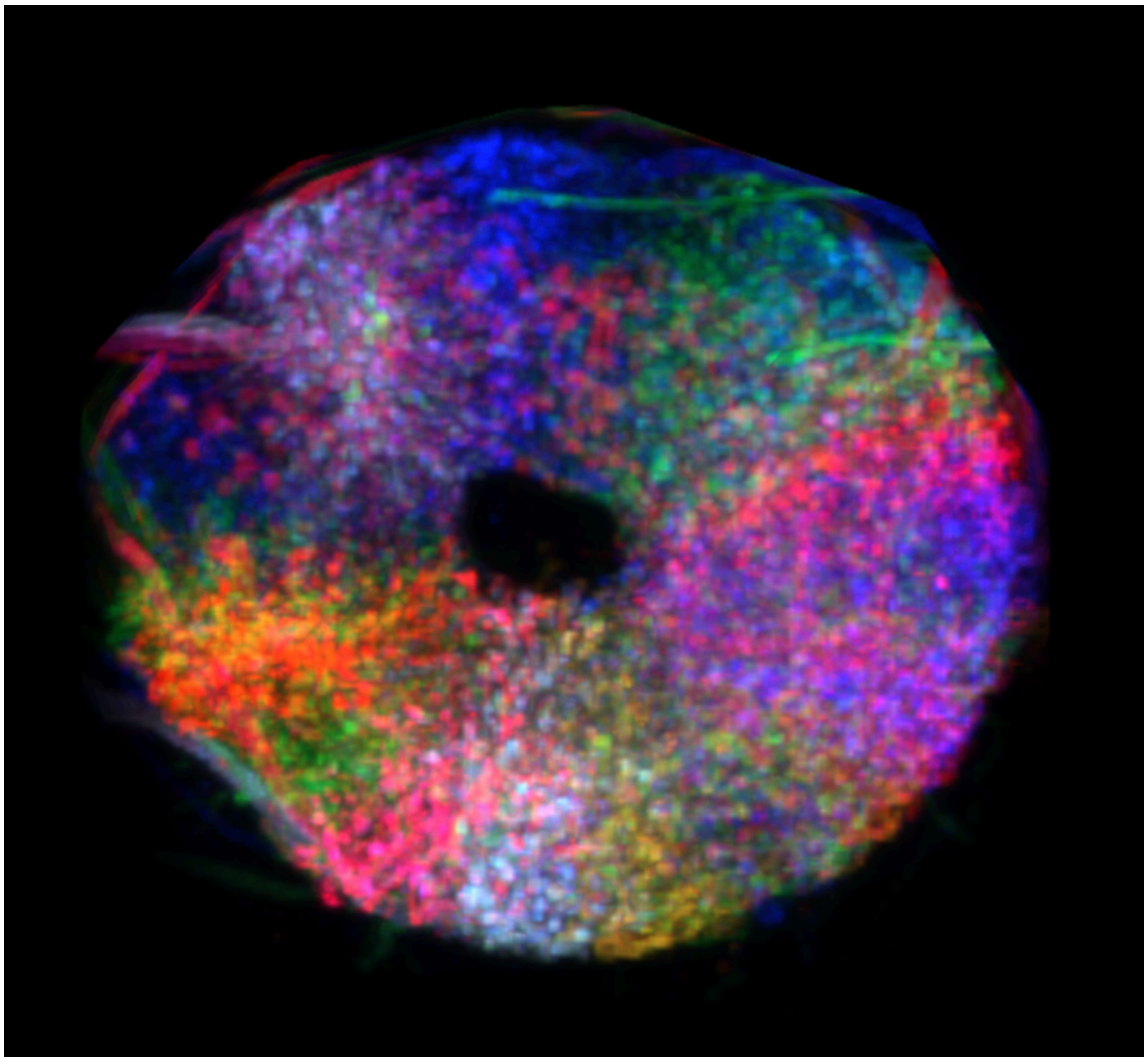
Figure 3: Architecture of the head direction cell model.

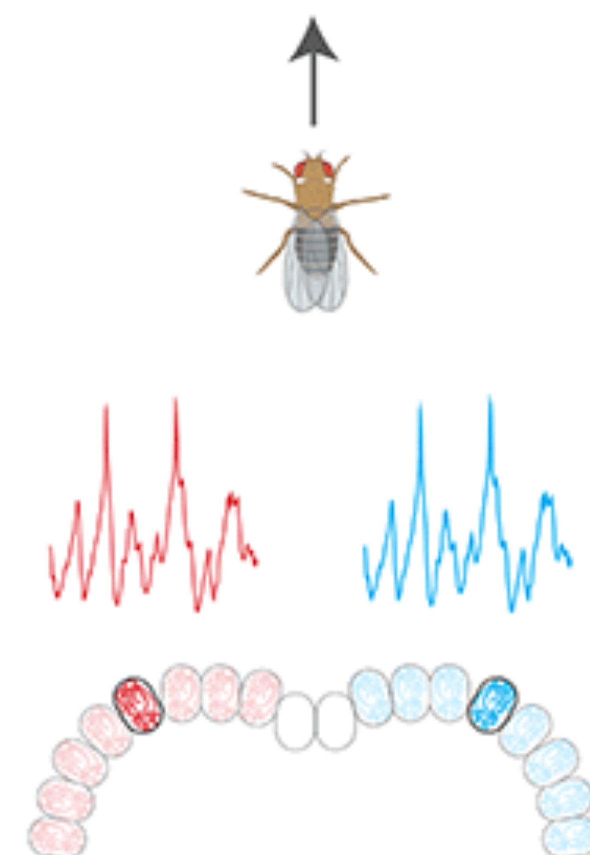
Skaggs, WE., et al. "A model of the neural basis of the rat's sense of direction." *Advances in neural information processing systems*. 1995.



Kim, Sung Soo, et al. "Ring attractor dynamics in the Drosophila central brain." Science 356.6340 (2017): 849-853.

2017: Found the ring network in *Drosophila* (fruit fly)!





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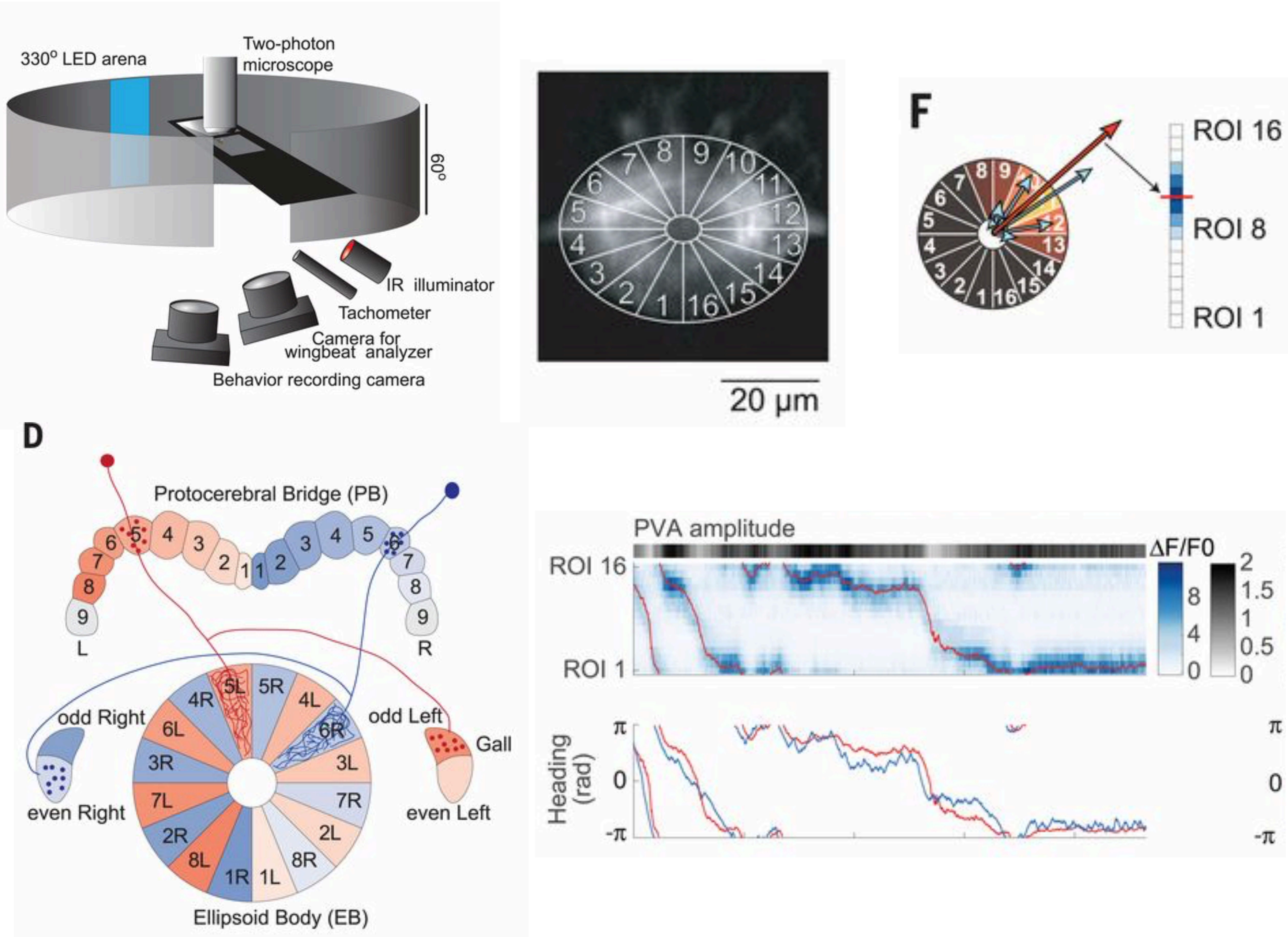
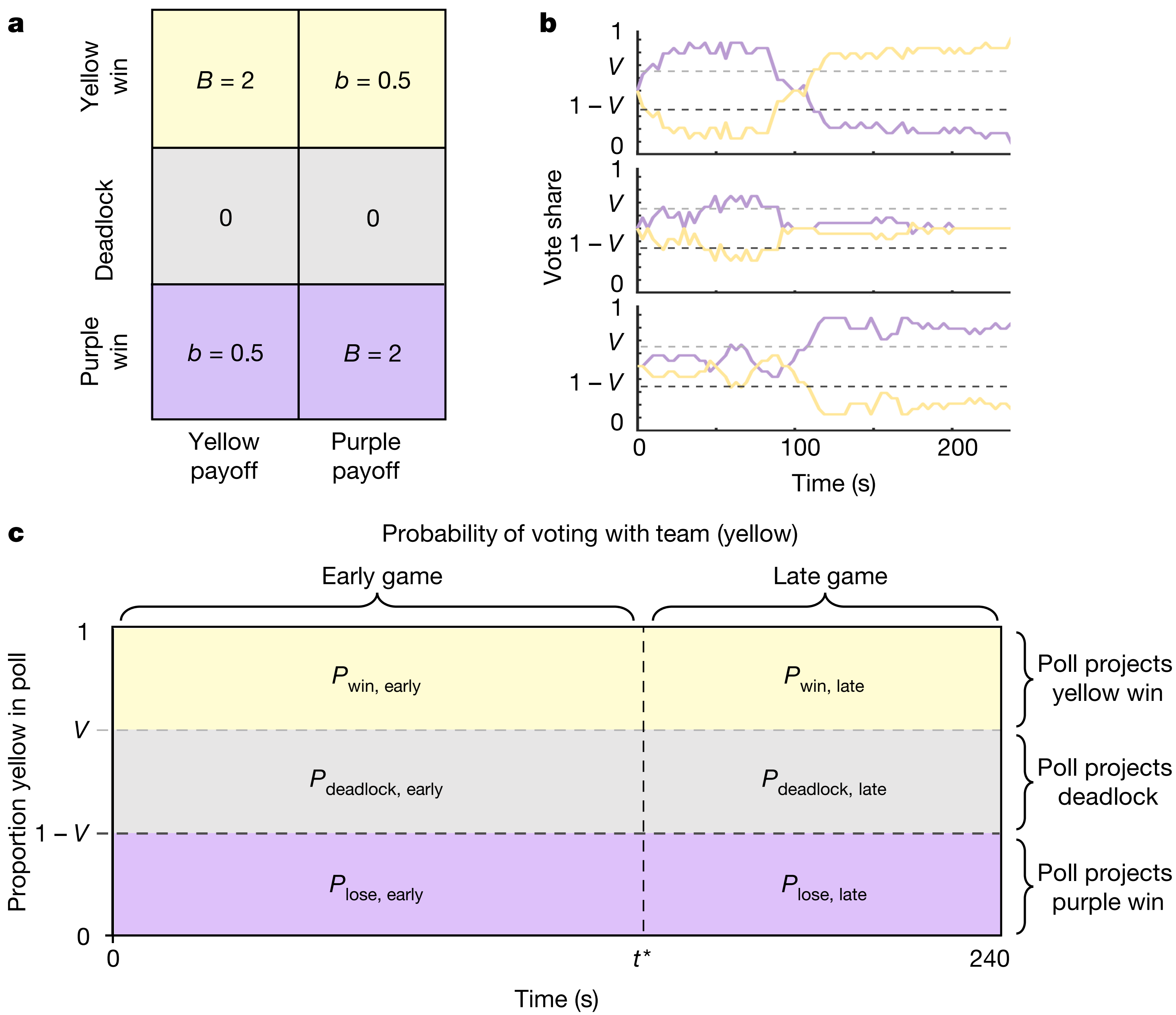
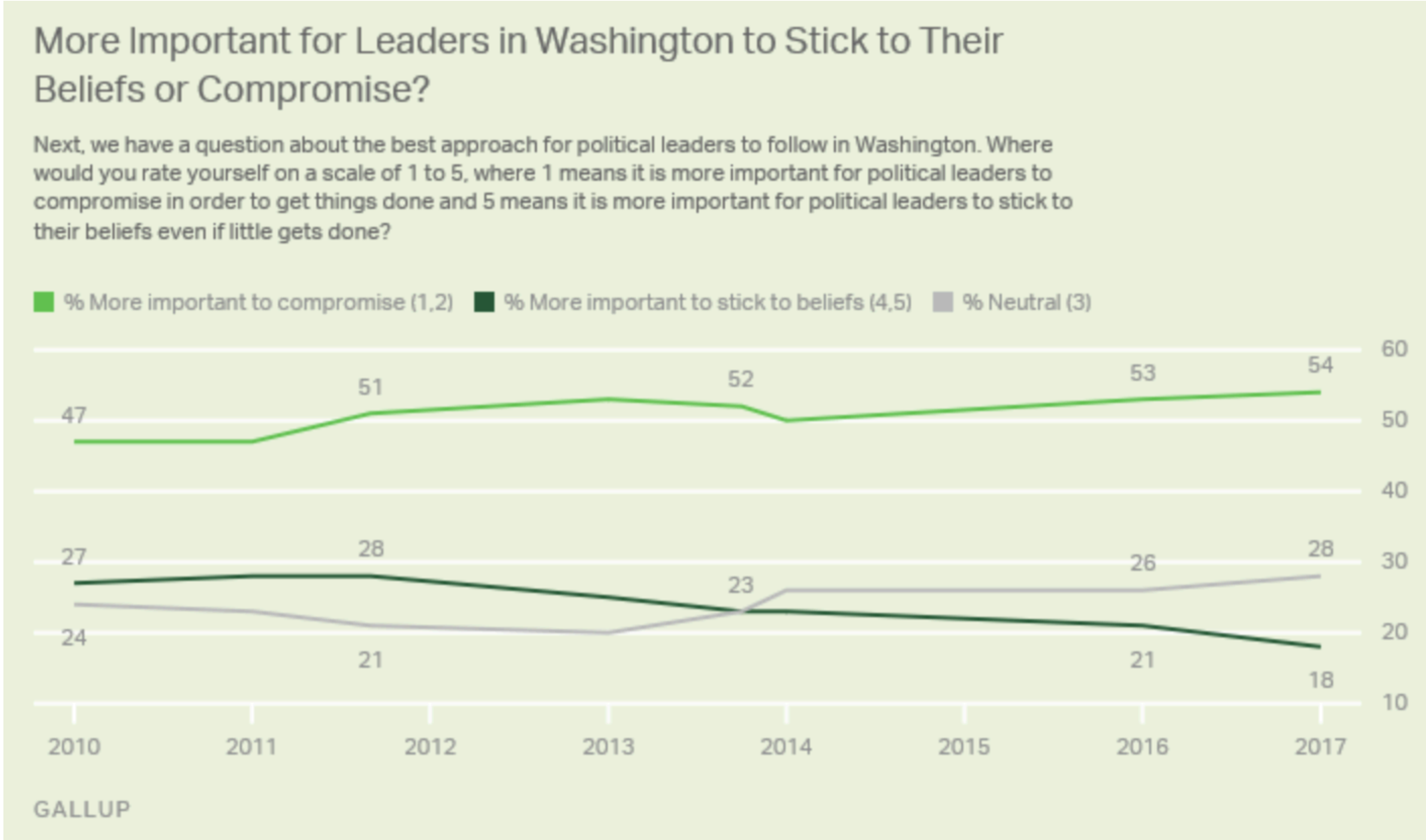


330° LED arena
Two-photon microscope
60°
IR illuminator
Tachometer
Camera for wingbeat analyzer
Behavior recording camera
8 9 10 7 11 6 12 5 13 4 14 3 15 2 1 16
20 µm
F
8 9 7 6 5 4 3 2 1 16 15 14 13 12
ROI 16
ROI 8
ROI 1
D
Protocerebral Bridge (PB)
9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9
L
R
odd Right
even Right
odd Left
even Left
Gall
5L 5R 4R 4L 6L 6R 3R 3L 7L 7R 2R 2L 8L 8R 1R 1L
Ellipsoid Body (EB)
PVA amplitude
ΔF/F0
ROI 16
ROI 1
8 4 0
2 1.5 1 0.5 0
Heading (rad)
π
0
-π

Example: information gerrymandering



Stewart, Alexander J., et al. "Information gerrymandering and undemocratic decisions." Nature 573.7772 (2019): 117-121.



More Important for Leaders in Washington to Stick to Beliefs or Compromise?

	2013	2014	2016	2017
	%	%	%	%
Republicans/Republican leaners				
More important to stick to beliefs	33	32	22	23
Neutral	25	31	29	33
More important to compromise	42	37	48	44
Democrats/Democratic leaners				
More important to stick to beliefs	19	13	20	12
Neutral	17	18	24	24
More important to compromise	63	67	56	62

GALLUP

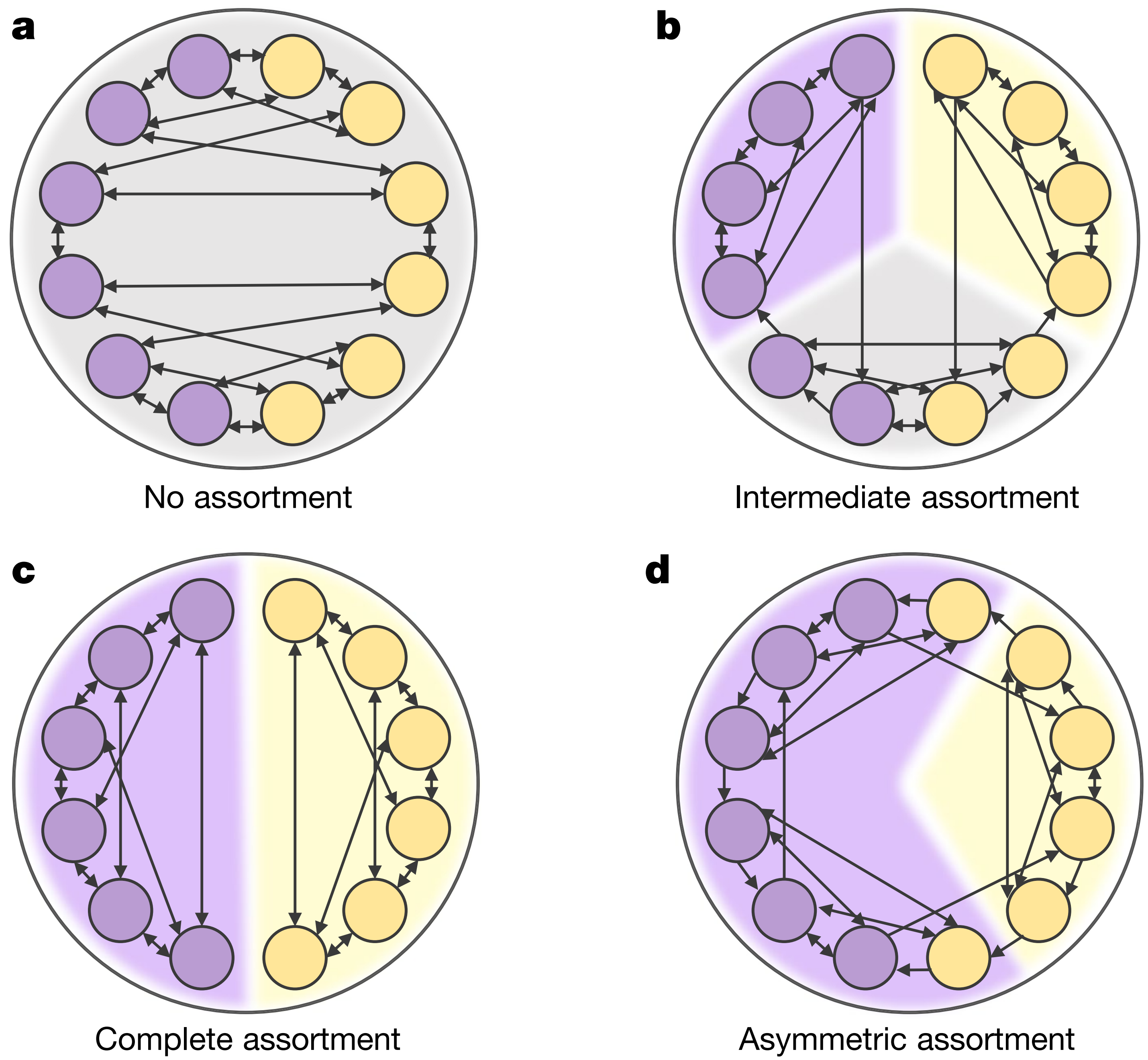
https://news.gallup.com/poll/220265/americans-favor-compromise-things-done-washington.aspx

Electoral Gerrymandering

Consider 24 people, 12 favoring the Purple party
and 12 favoring the Yellow party

Stewart, Alexander J., et al. "Information gerrymandering and undemocratic decisions." Nature 573.7772 (2019): 117-121.

Network influence assortment



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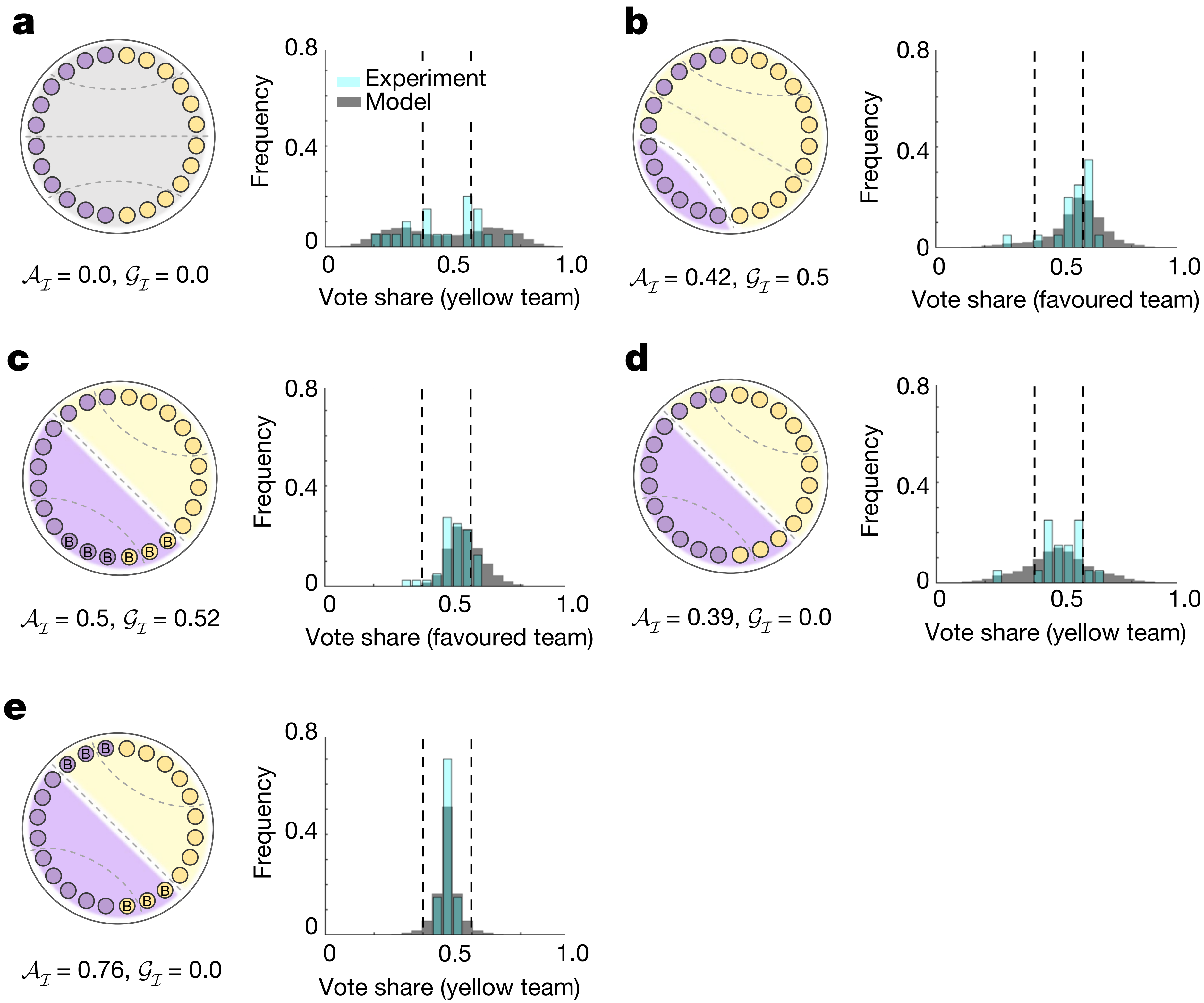
Experimental data



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Examples

- Percolation on a network
- Diffusion on a network (movement, etc.)
- Regulatory relationships in cells (levels of gene activity, protein concentrations, etc.)
- Ecological relationships (species populations)
- Coupled oscillators (e.g. fireflies etc)
    - https://ncase.me/fireflies/
    - PyCX example code

Dynamics **of** networks

- Things to consider
  - How do we add/remove nodes?
  - How do we add/remove edges?
- Dynamics of networks can often be framed as dynamics on networks where we activate/inactivate nodes/edges in a super-network
  - E.g. sexual network partnerships

Dynamics **of** networks

- Often depends on the question at hand—often the rules for changing network structure are often question and system specific

- Random graph generators from earlier can also be thought of as dynamics of networks

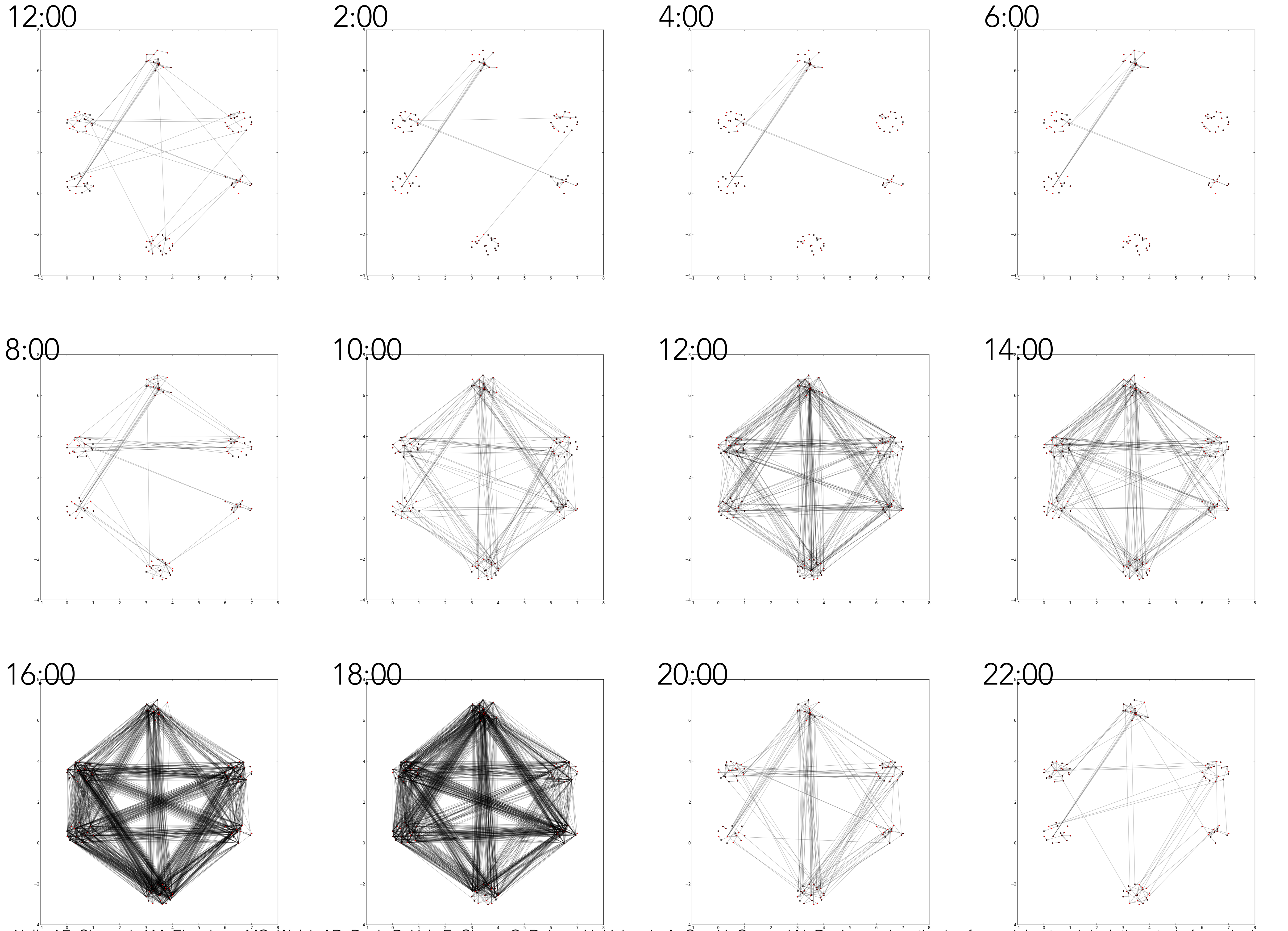
    - Erdös-Renyi

    - Small world

    - Preferential attachment

Dynamics **of** networks

- Dynamic empirical networks - contact networks, travel networks, ecological networks, trade networks, social media networks, etc.



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Examples

- Evolution of gene regulatory and metabolic networks
- Self organization, adaptation of food webs
- Social network formation and change, growth of collaboration and citation networks
- Global economic relationships, trade, diplomacy, etc.
- Growth of infrastructure networks (power grids, sanitation, traffic, railways, internet)
- Many of these are potentially adaptive networks

Rail network

Internet fiber cable network

https://en.wikipedia.org/wiki/Rail_transportation_in_the_United_States

https://www.technologyreview.com/s/540721/first-detailed-public-map-of-us-internet-backbone-could-make-it-stronger/

For next time…

- Reading
  - Sayama Chapter 16
  - Think Complexity Chapters 4 & 5