Model & likelihood vs. code

Model

ODE model:

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

where S, I, R are the fractions of the population that are susceptible, infectious, and recovered

Measurement eqn

suppose our data is a measure of prevalence, but not every infection is tested/reported.

Then we measure:

Code

```
SIRode <- function(t, x, params){
  S = x[1]
  I = x[2]
  R = x[3]

  b = params[1]
  g = params[2]

  dSdt = -b*S*I
  dIdt = b*S*I - g*I
  dRdt = g*I

  list(c(dSdt, dIdt, dRdt))
}
```

$$y = I \times N \times k$$

fraction infections × total population × reporting fraction

total number infections

= the reported # of infections.

But, to simplify things, we'll combine k and N into one parameter, K

$$y = I \cdot K$$

Initial Conditions

We can use the measurement equation to roughly estimate the initial condition for I:

$$I(0) = y(0)/K$$

$$I(0) \approx data(0)/K$$

$$S(0) = 1 - I(0)$$

$$R(0) = 0$$

```
yfun = function(odeSim, params){odeSim[,3]/params[3]}
```

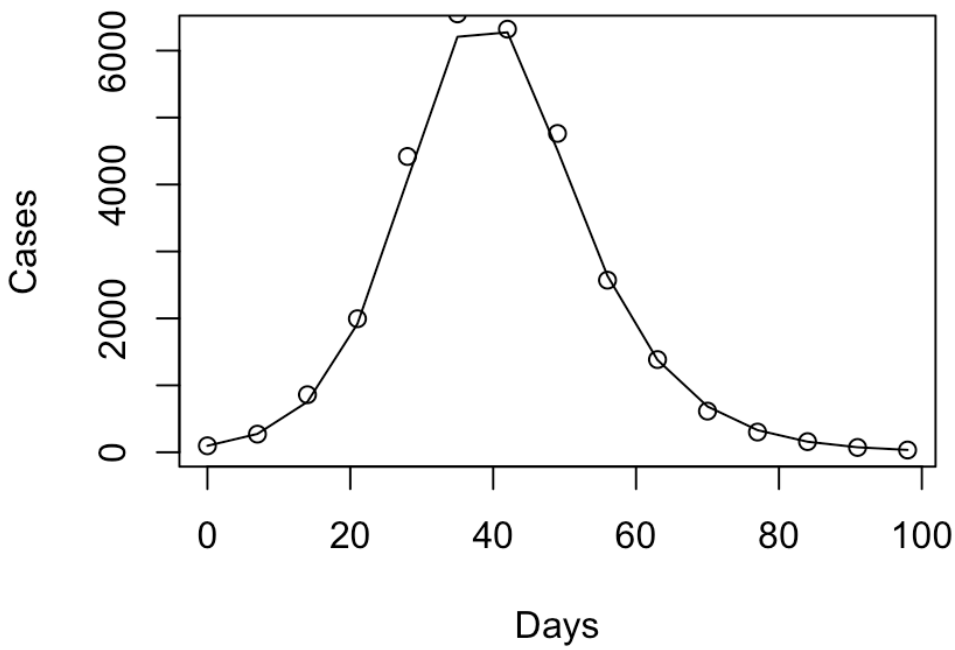
```
x0fun = function(cases,params) {
  x0 = c("S0" = 1-(cases[1]*params[[3]]),
         "I0" = cases[1]*params[[3]],
         "R0" = 0)
  }
```

Simulate the model

```
times = c(0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98)
cases = c(97, 271, 860, 1995, 4419, 6549, 6321, 4763, 2571, 1385, 615, 302, 159, 72, 34)
dataset = cbind(times, cases)

params = c('beta'=0.4,'gamma'=0.25, 'kappainv'=1/80000)

xinit <- ode(x0fun(cases,params), times, SIRode, params, method='ode45')
plot(times, yfun(xinit,params), type='l')
points(dataset)
```



likelihood

negative log likelihood

$$-LL = \text{Poisson}(z, y)$$

data: z, mean: y

equivalent to:

$$-LL = \sum_{t=1}^{n} y_t + \sum_{t=1}^{n} z_t \log y_t$$

```
SIRML=function(params,times,data){
  params = abs(params)

  # Simulate model
  xcurr = ode(x0fun(data,params), times, SIRode, params, method='ode45')

  # Measurement equation
  y = yfun(xcurr,params)

  # Negative Log Likelihood (NLL)
  NLL =  sum(y) - sum(data*log(y)) # Poisson ML

}
```