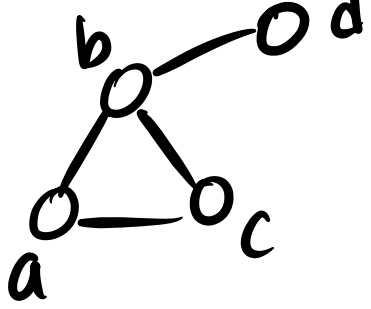
Network likelihood example 3/2023

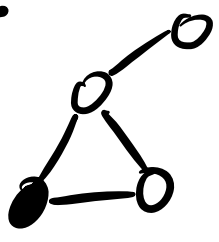
Suppose we have a teeny tiny network:

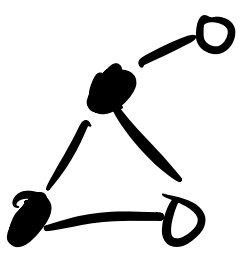


(labels so we can talk about individual nodes later)

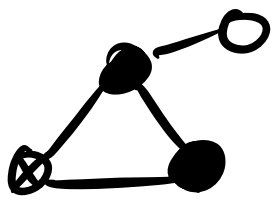
And we have an SIR-process on the network, with data on who gets infected when:

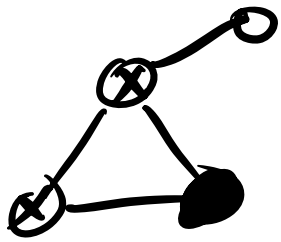
Data:

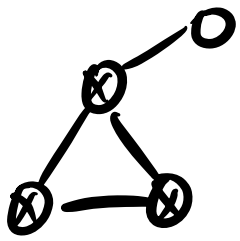
t_0: 

t_1: 

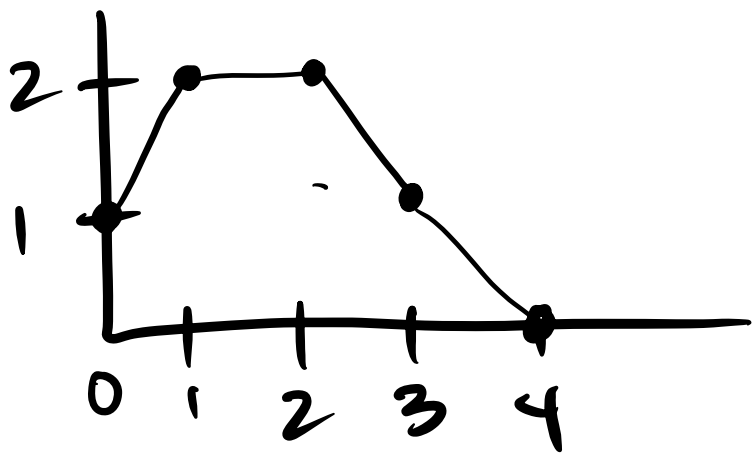
○ = susc.
● = inf.
⊗ = rec

t_2: 

t_3: 

t_4: 

Our epidemic curve looks like:



Okay, so what's the likelihood fnction?

It should be the probability of observing this data given our parameter values – we can break it down like this:

$$\mathcal{L} = \mathcal{L}_1 \mathcal{L}_2 \mathcal{L}_3 \mathcal{L}_4$$

where $\mathcal{L}$ is our total likelihood and $\mathcal{L}_i$ is the contribution for day i. Then, with our usual SIR model, we have:

$$\mathcal{L}_1 = p_i (1 - p_i)(1 - p_r)$$

p_i: probability a infects b

$(1 - p_i)$: probability a does not infect c

$(1 - p_r)$: probability a does not recover

$$\alpha_2 = p_r(1-p_r)(1-(1-p_i)^2)(1-p_i)$$

a recovers — b doesn't recover — either a or b infects c — b does not infect d

$$\alpha_3 = p_r(1-p_r)(1-p_i)$$

$$\alpha_4 = p_r$$

For fun: think about how this would change if we didn't have data on who was infected but just the total each day (the epidemic curve)! And what if we didn't know the network structure?